MATH 2260H Bolyai-Gerwein Theorem III

Back to our 6-step proof of

<u>Thm:</u> Two polygons are scissors-congruent if and only if they have the same area.

The steps so far:

1) Every polygon can be dissected into triangles. (Only did it for convex and star-shaped polygons.)
2) Any triangle can be dissected and the pieces reassembled into a ~~square~~ rectangle.
3) Any rectangle can be dissected and the pieces reassembled into a square.
4) Any two squares can be ~~reassemble~~ dissected and reassembled into a single larger square.

On to steps 5 & 6 ...

5) Any polygon can be dissected and the pieces reassembled into a square (of equal area).

(There should be an induction here, but it's pretty obvious.)

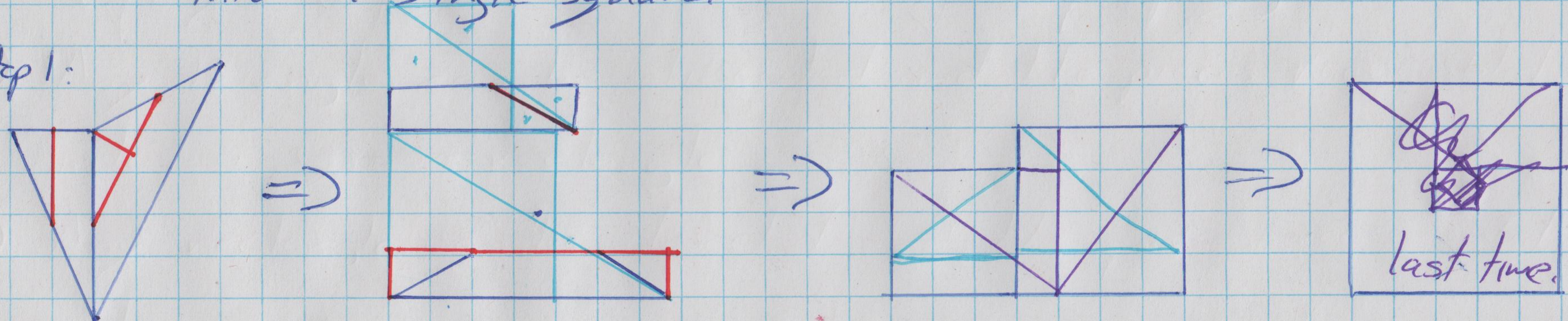
Given a polygon, dissect it into triangles, say n of them. (Step 1)

Apply Step 2 to dissect each triangle into rectangles.

Apply Step 3 to dissect each rectangle into squares.

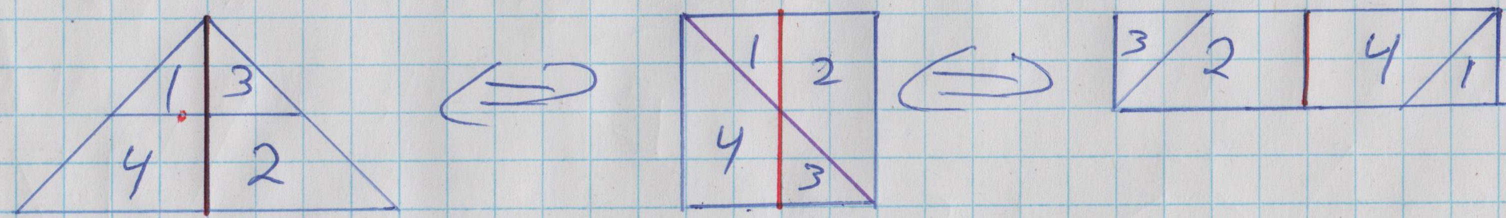
Apply Step 4 repeatedly to combine the n squares into a single square.

After step 1:



Keep track of the cuts at each stage [superimposing them] in order to get final (direct) dissection.

6) Given any two polygons of equal area, dissect each one into a square. The squares have equal area, so they are congruent. Superimpose the dissected squares over each other to get a dissection that can be used to directly dissect one polygon and reassemble it into the other.



How does the iff happen here?

Built in because we can do steps 6 & 5.

& squares of equal area are congruent.