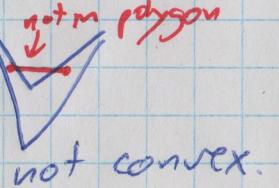
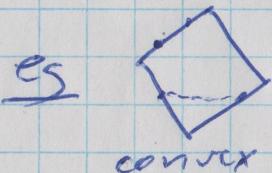


Recall: A dissection of a polygon is a partition of the polygon into smaller polygons. Two polygons are scissors-congruent if each can be dissected into the same pieces as the other. (i.e. One can pair off congruent pieces, one from each dissection.)

Theorem: Two polygons are scissors-congruent if and only if they have equal areas.

① Step 1: Every polygon can be dissected into triangles.
Tricky to prove for all polygons, but two common cases are easy.

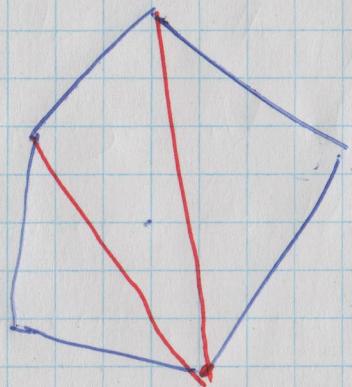
1) Polygon is convex - i.e. every line segment joining two points on the perimeter is part of the polygon



(2)

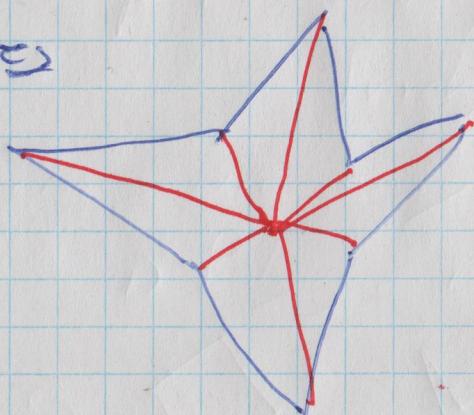
For a convex polygon, simply connect one vertex to all the others, to partition the polygon into triangles.

ex



- 2) Polygon is star-shaped - ie there is some point in the interior of the polygon such that the line segments joining it to the vertices are all in the polygon.

ex



Connect this point to the vertices to get a partition of the polygon into triangles.

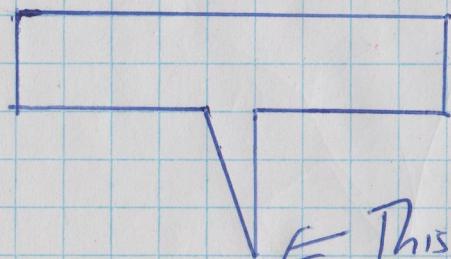
(3)

3) For non-star-shaped polygons it can get pretty hard. One can do an induction on the number of vertices n ($n \geq 3$) in the polygon, but you have all sorts of problems at the induction step:

Idea: cut the polygon along a line joining two (non-adjacent) vertices into two pieces & apply the H to each.

We'll leave this general case to you.

Difficulty: if you pick a vertex, you might not be able to connect it to a non-adjacent one by a line-segment that runs in the polygon

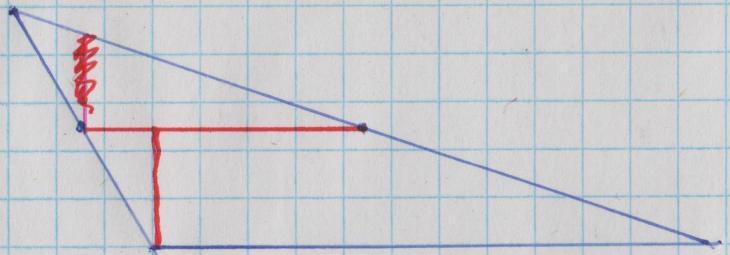


→ This vertex does not connect nicely. Pick another? How?

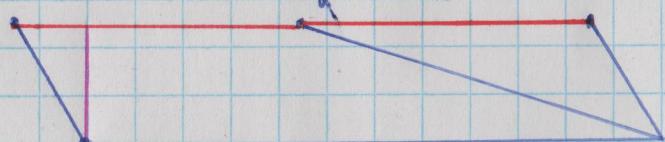
so we'll assume step 1) is complete.

(9)

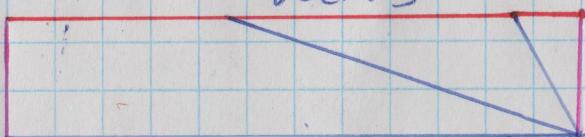
Step 2). We can dissect any triangle and reassemble it into a rectangle.



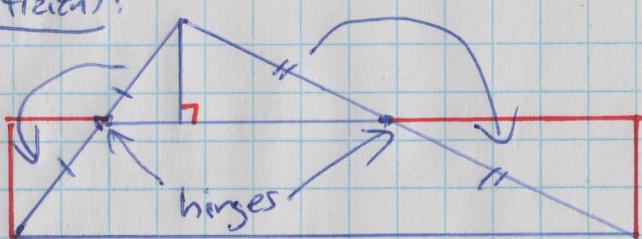
↓ This makes a parallelogram



↓ Makes a rectangle.



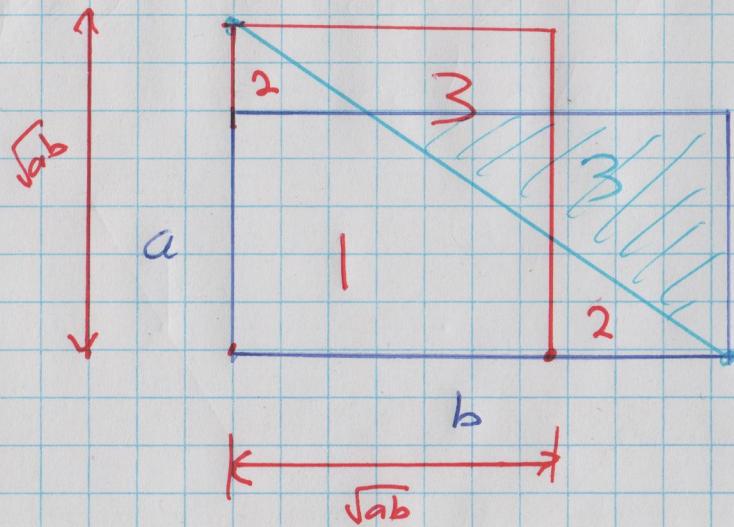
More efficient:



Pick a side to be the base and connect the midpoints of the other two sides. Cut along this segment & make a parallelogram & the cut an end off and attach it at the other end to make a rectangle

Step 3) - Any rectangle can be dissected into a square.

(5)



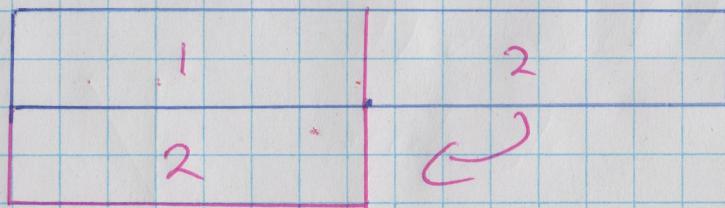
Suppose the rectangle has sides of length a & b , where $a < b$.

Then $a < \sqrt{ab} < b$. A square of sides \sqrt{ab} will have the same area.

Then partition as in the diagram.

Two caveats: a) This only works so long as $b \leq 4a$.

If $b > 4a$,

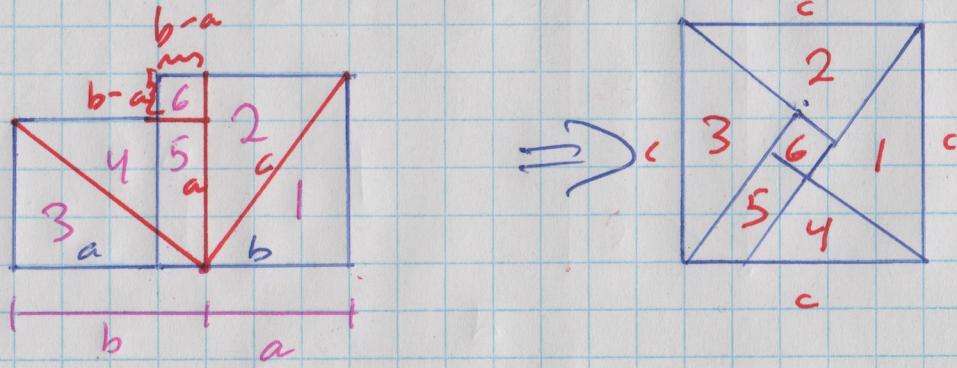


cut it in half & stick
& repeat as
necessary until
 $b \leq 4a$.

b) We can actually ^{construct} find a line segment of length \sqrt{ab} in Euclid's system. (This is a consequence of Prop. II-6, which see.)

⑥

Step 4) - Any two squares can be dissected and reassembled into a single larger square.



Steps 5) & 6) next time!