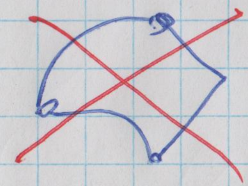
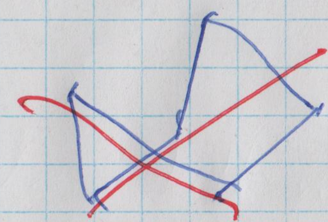
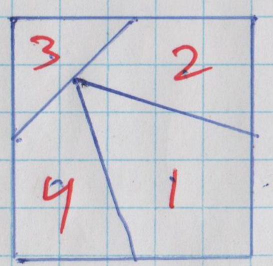
Note: In what follows a polygon is a finite 2-D shape which straight sides and is not self-overlapping.



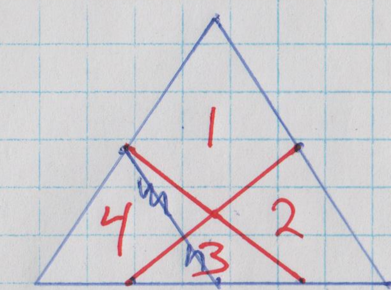


Def'n: A dissection of a polygon is a partition of the polygon into smaller ~~ne~~ polygons (not overlapping with each other).



A dissection of a square into four pieces.

A dissection of an equilateral triangle into four pieces.



The four pieces are the same!

What do we need to make such a thing happen?

At minimum, the original polygons have to have equal areas.

<u>Def'n:</u> Two polygons are scissors-congruent if each can be decomposed into finitely many [same number!] ~~of~~ polygons which are pairwise congruent.

[TE each ~~of~~ can be taken apart into the same pieces.]

<u>Bolyai-Gerwein Thm:</u>

Two polygons are scissors-congruent if and only if they have equal areas.

Farkas Bolyai proved this in 1833 & Paul Gerwein in 1835. It was first proven by William Wallace in 1807.

eg The unit square and the equilateral triangle with sides of length $\frac{2}{\sqrt[4]{3}} = 2 \cdot 3^{-1/4}$ are scissors-congruent because they have equal areas.

Plan of the proof:

1. Show that every polygon can be dissected into triangles.
2. Show that every triangle can be dissected and reassembled into a rectangle.
3. Show that every rectangle can be dissected and reassembled into a square.
4. Show that any two squares can be dissected and reassembled into a single square.
5. (By induction) Show that any polygon can be dissected into pieces that can be reassembled into a square of equal area.
6. Show that any two polygons of equal area are scissors-congruent.

Some details next time!