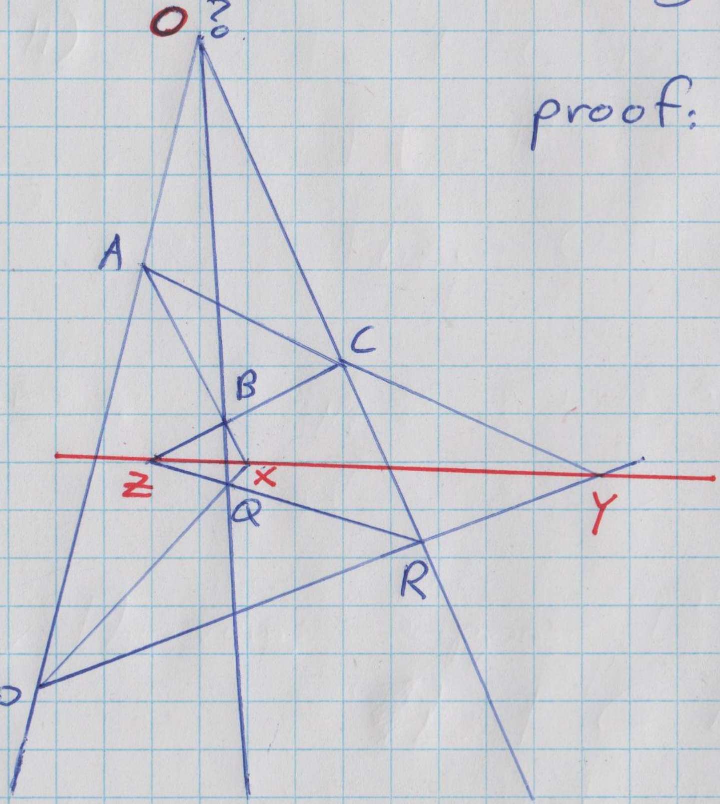
MATH 2260H Desargues' Thm. II



proof: ($\Leftarrow$) Suppose we have ΔABC & ΔPQR s.t. $X = AB \cap PQ$, $Y = AC \cap PR$, & $Z = BC \cap QR$.

[To show: AP, BQ, & CR are concurrent.]

Let $O = PA \cap QB$. Now observe, that by Menelaus' Thm.

1) ΔXAP & O, B, Q collinear

$$\Rightarrow \frac{PO}{OA} \cdot \frac{AB}{BX} \cdot \frac{XQ}{QP} = -1$$

2) $\Delta X\cancel{A}Y$ & B, C, Z are collinear

$$\Rightarrow \frac{XB}{BA} \cdot \frac{AC}{CY} \cdot \frac{YZ}{ZX} = -1$$

3) ΔXPY & R, Q, Z are collinear

$$\Rightarrow \frac{XQ}{QP} \cdot \frac{PR}{RY} \cdot \frac{YZ}{ZX} = -1$$

4) Since we have ΔPAY and

$$\frac{\left(\frac{PO}{OA} \cdot \cancel{\frac{AB}{BX}} \cdot \cancel{\frac{XQ}{QP}}\right) \cdot \left(\cancel{\frac{XB}{BA}} \cdot \frac{AC}{CY} \cdot \cancel{\frac{YZ}{ZX}}\right)}{\cancel{\frac{XQ}{QP}} \cdot \frac{PR}{RY} \cdot \cancel{\frac{YZ}{ZX}}} = \frac{PO}{OA} \cdot \frac{AC}{CY} \cdot \frac{RY}{PR} = \frac{PO}{OA} \cdot \frac{AC}{CY} \cdot \frac{YR}{RP}$$

$$= \frac{(-1)(-1)}{(-1)} = -1$$

, so by Menelaus' Thm, O, C, R are collinear. Thus PA, QB, & CR are concurrent in the point O. //