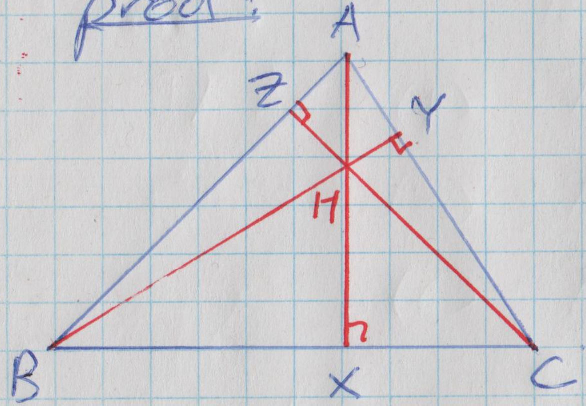
MATH 2260H Triangles and their centres V - More about the orthocentre

2021-03-14

Def'n: An *orthocentric system* (or *orthocentric quadrangle*) is a set of four points in the plane, such that each is the orthocentre of the triangle formed by the other three.

Prop.: Suppose $\triangle ABC$ is a triangle with orthocentre H. Then A, B, C, H form an orthocentric system.

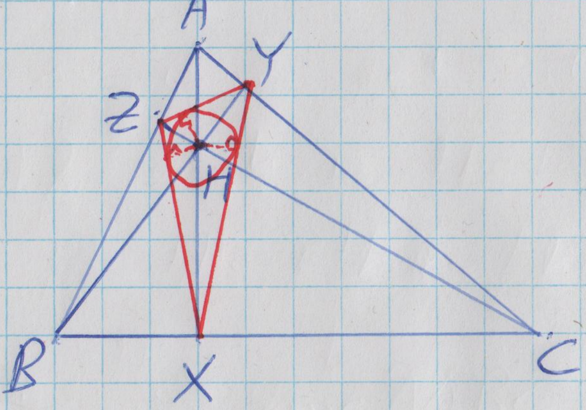
proof:



Obviously (by def'n) H is the orthocentre of $\triangle ABC$.

Consider $\triangle ABH$. CZ (which extends H past H) is perpendicular to AB. BH is perpendicular to AC and meets AC at Y. AH is perpendicular to BC & meets it at X. It follows that C is the orthocentre of $\triangle ABH$. Similarly B is the orthocentre of $\triangle ACH$ & A — " — $\triangle BCH$. //

Prop: Suppose H is the orthocentre of ΔABC, with X, Y, Z being the feet of the altitudes from A, B, C, resp.
Then $|AH|\cdot|HX| = |BH|\cdot|HY| = |CH|\cdot|HZ|$.



proof: Think about it! //

Prop: Suppose H is the orthocentre of ΔABC, with X, Y, Z being the feet of the altitudes from A, B, C, resp.
Then H is the incentre of ΔXYZ.
(Note that this means that the altitudes of ΔABC are the angle bisectors of ΔXYZ.)

proof: Think about it! //

Next time we'll do one of these...