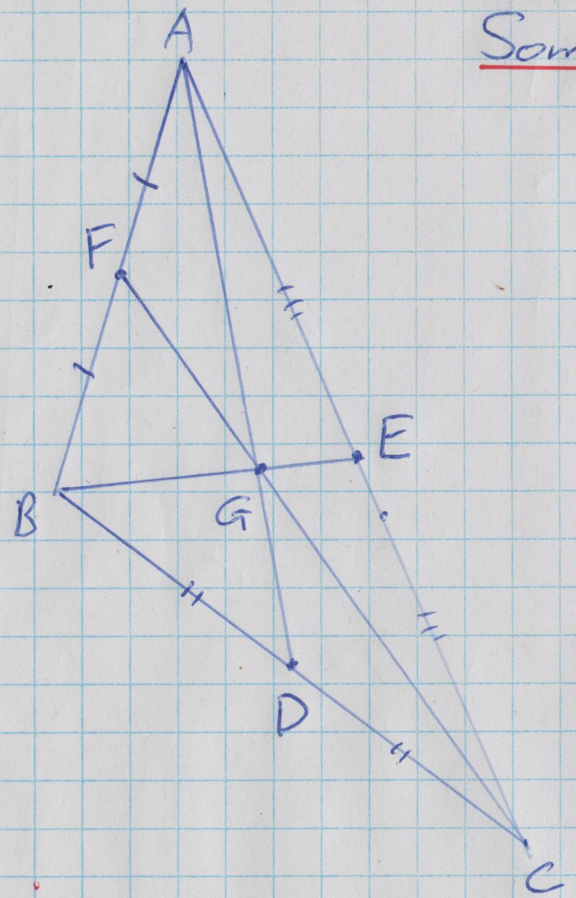
MATH 2260H 2021-03-09

Triangles and their centres III – Some tidbits about the medians and the centroid



Prop.: The medians of a triangle divide it into six smaller triangles of equal area.

Notation: $|\triangle ABC|$ = area of $\triangle ABC$

proof: Let D, E, F be the midpoints of sides BC, CE, and AB, resp., of $\triangle ABC$, and let G be the centroid of $\triangle ABC$. Observe that $\triangle AGF$ and $\triangle BGF$ have equal area because they have equal bases, $|AF| = |BF|$, and equal heights since the vertex G is common to them and AF and BF are both part of AB. Similarly, $|\triangle BGD| = |\triangle CGD|$ and $|\triangle CGE| = |\triangle AGE|$.

Similar arguments can be applied to other subtriangles,

$|\Delta ADB| = |\Delta ADC|$, and $|\Delta BGD| = |\Delta CGD|$ (as previously observed).

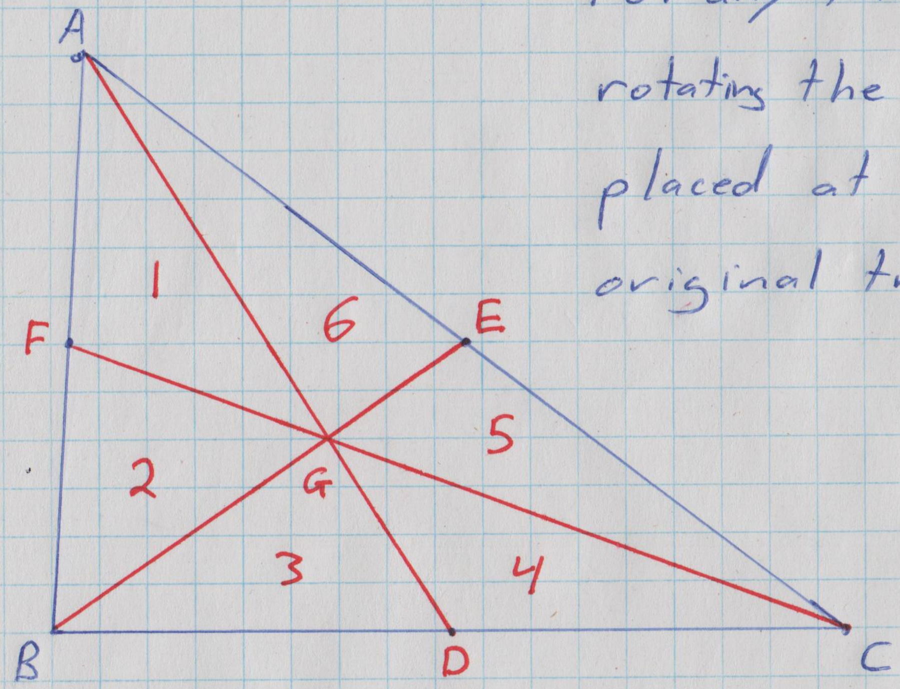
But then
$$
\begin{aligned}
|\Delta ABG| &= |\Delta ABD| - |\Delta BGD| \\
&= |\Delta ADC| - |\Delta CGD| \\
&= |\Delta AGC|,
\end{aligned}
$$

so $|\Delta AFG| = \frac{1}{2}|\Delta ABG| = \frac{1}{2}|\Delta AGC| = |\Delta AEG|$.

Similar arguments show that $|\Delta BFG| = |\Delta BDG|$

& $|\Delta CDG| = |\Delta CEG|$.

∴ the six subtriangles created by the medians have equal areas. //



<u>Theorem:</u> (Lee Sallows, 2014)

For any triangle dissected along its medians, rotating the smaller triangles about three hinges placed at the midpoints of the sides of the original triangle yields three congruent triangles.

proof: "Elementary." Left to you! //