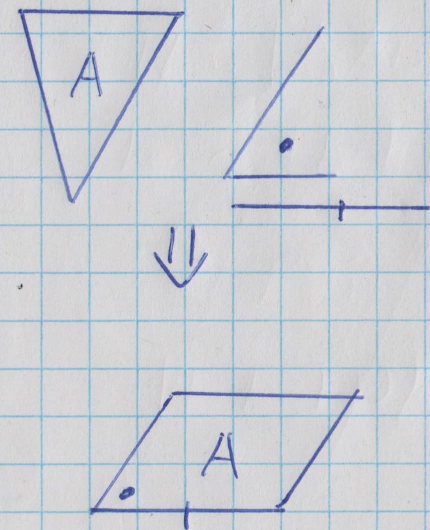
MATH 2260H Propositions I-44 to I-48

or, the Pythagorean Theorem (I-47)

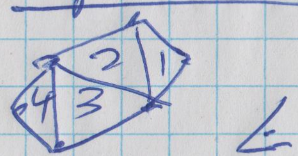
Prop. I-44: Given a triangle and a line segment and an angle, construct a parallelogram with the line segment as one side, and an internal angle equal to the given one, and equal in area to the triangle.

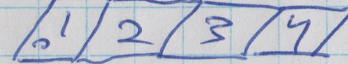


proof: Look it up in the *Elements*. //

* Important to Euclid since he does areas in terms of parallelograms, but less so to us.

Prop I-45: Given a polygon and an angle, construct a parallelogram equal in area to the polygon, and with an internal angle equal to the given one.



proof: Divide up the polygon into triangles and use I-44 repeatedly. (Look it up...) 

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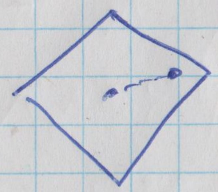
Note: Any polygon can be divided up into triangles, but this takes some effort to prove if the polygons don't have to be convex.



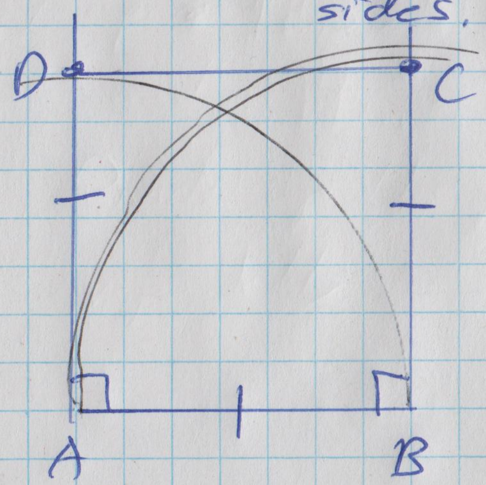
convex

non-convex

Euclid only does the case of a convex quadrilateral.

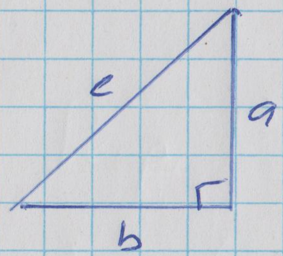
Proposition I-46: Given a line segment, construct a square with that line segment as one of the sides. [This a counterpart to I-1.]

proof:



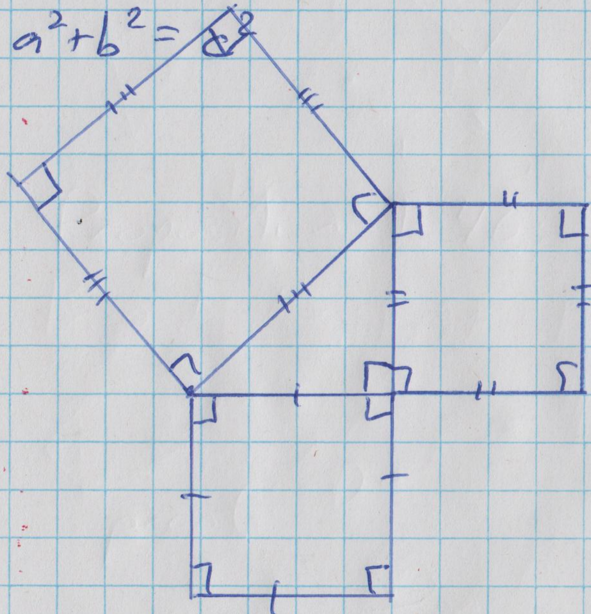
Given AB, construct perpendicular lines to AB at A & B. Draw circles with centres A & B & radius AB to mark off points C & D on these lines and connect up C & D. This gives a Saccheri quadrilateral with all sides equal, so (using Post. V) all angles are right. //

Prop I-47: (Pythagorean Thm.)

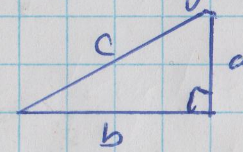


In a right-angled triangle, the square on the side subtending the right angle is equal to the sum of the squares on the other two sides.

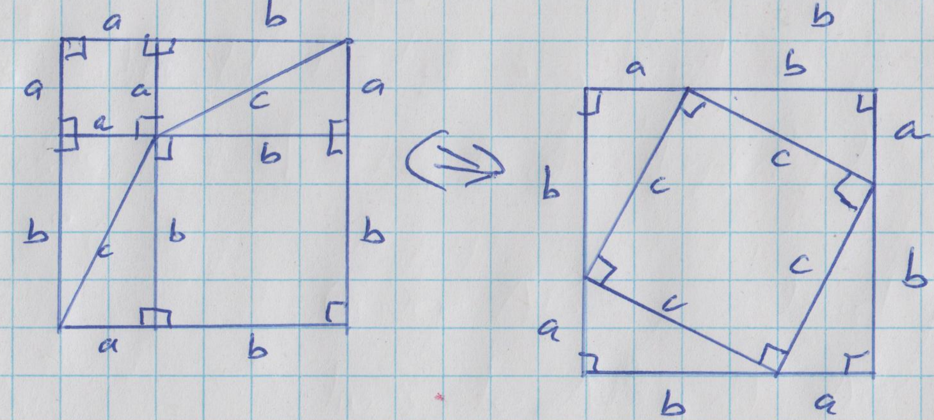
$a^2+b^2=c^2$



proof: We'll do a simpler proof than Euclid's. Let



be our triangle



$$(a+b)^2 = a^2+b^2 + \frac{4ab}{2}$$ ← area of the four triangles.

$$= c^2 + \frac{4ab}{2}$$

$$\Rightarrow a^2+b^2=c^2.$$

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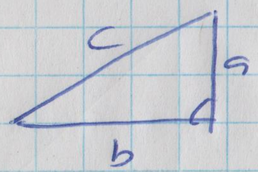
Prop. I-48: (Converse to the Pythagorean Theorem)



$a^2+b^2=c^2$

$\Rightarrow \angle = \llcorner$

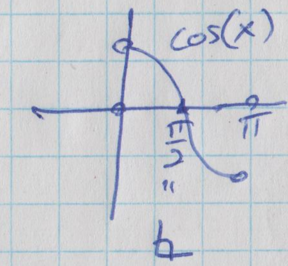
If the square on one side of a triangle is equal to the squares on the other two sides, then the triangle has a right angle opposite the first side.

proof: By the Law of Cosines,

$$c^2 = a^2+b^2 - 2ab\cos(\angle) \quad = a^2+b^2$$

$$\Rightarrow 2ab\cos(\angle) = 0 \quad \text{ie} \quad \cos(\angle) = 0$$

$$\Rightarrow \angle = \llcorner$$



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Coming to the future lectures: circles & related properties, especially of chords & their extensions & related angles.

[We'll be diverging from Euclid from here on in.]