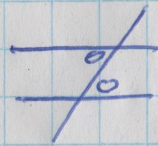
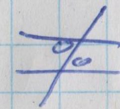
MATH 2260H Propositions I-30 to I-32, wherein we meet the most popular equivalents of Postulate V.

Recap:

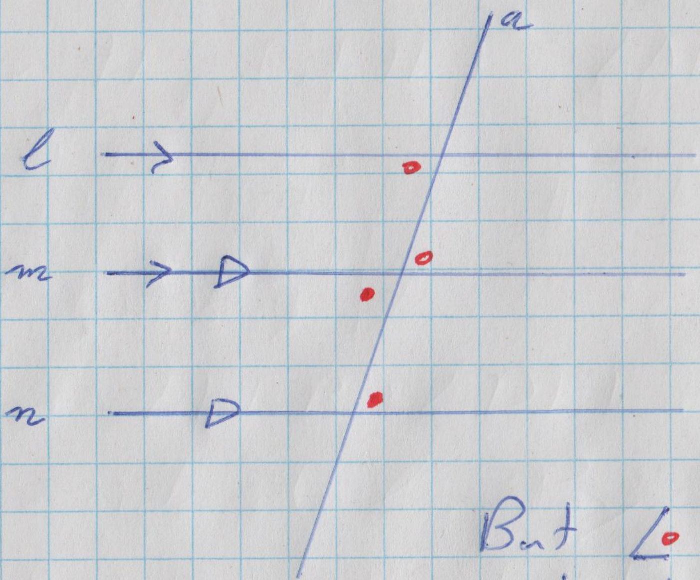
2-Thm.

I-27  ⇒  Needs only Post. I-IV (⊖ S&A).

I-29  ⇒  Requires Post. V as well.

Prop. I-30: Given three lines l, m, n such that $l \parallel m$, and $m \parallel n$, we also have $l \parallel n$. ["Parallel" is a transitive relation.]

proof:

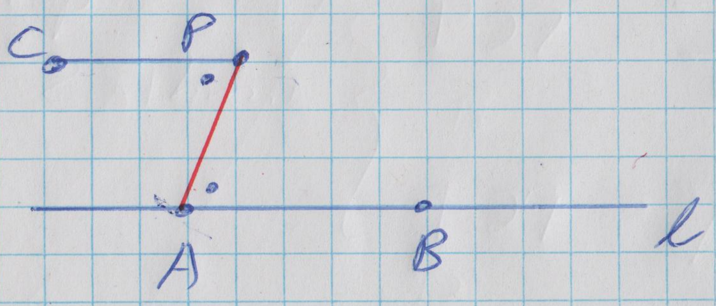


Suppose a falls across all three of l, m, and n. Since $l \parallel m$, a makes alternate interior angles $\angle^{\circ}$ equal (I-29), & similarly $m \parallel n$ means a makes alt. int. angles $\angle^{\bullet}$ equal.

But $\angle^{\circ} = \angle^{\bullet}$ at line m by the Opposite Angle Thm. (I-15), so by I-27, $l \parallel n$ ... //

Prop. I-31 Given a point P and a line ℓ, there is a line m through P parallel to m. [Playfair's Axiom - a.k.a. Post V' - without the uniqueness of m.]

proof.



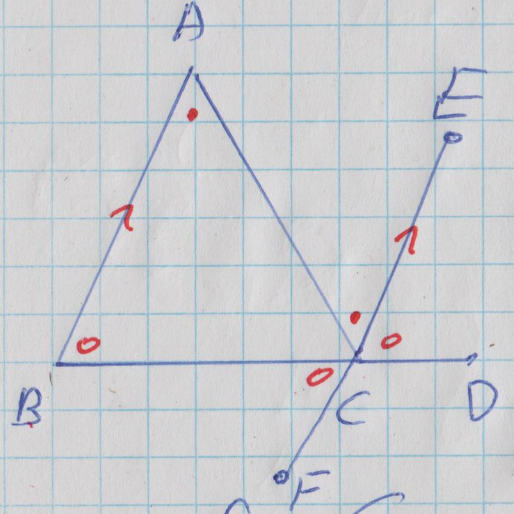
Pick a point A and another point B on ℓ. Draw AP. Draw an angle ∠APC equal to ∠PAB at P (I-23) on the other side of PA from B.

Thus ∠APC = ∠PAB, so by I-27 CP is ∥ to ℓ.
[The line m that extends CP is the line we need.] //

Note: This only requires much that we have Postulates I-IV (+ A&S)...

Exercise: Showing that m is unique requires Post. V.

Prop. I-32: "For any triangle, if one of the sides is produced, then the external angle is equal to the sum of the two internal and opposite angles, and the sum of the three internal angles of the triangles is equal to two right angles."



proof: Given ΔABC "produce" BC past C to some point D.

To show: (1) $\angle ACD = \angle ABC + \angle BAC$ ✓

(2) $\angle ABC + \angle BCA + \angle BAC = 2\llcorner = \frown$

Use Prop. I-31 to draw a line FE through C which is parallel to AB. AC falls across AB and CE, so it makes alternate interior angles equal, ie $\angle BAC = \angle ACE$. BC fall across AB and CE so $\angle ABC = \angle BCF$. By Opposit Angles Thm. (I-15) $\angle ECD = \angle BCF = \angle ABC$. $\therefore \angle ACD = \angle ACE + \angle ECD = \angle BAC + \angle ABC$.

Also, $\frown = \angle BCD = \angle ACB + \angle ACE + \angle ECD = \cancel{\angle BAC} \angle ACB + \angle BAC + \angle ABC$. //

In fact the assertions that, respectively,

the sum of the interiors angles of a triangle is a straightangle,

and

through a given point there is an unique ~~parall~~ line parallel to a given line,

are each equivalent to Postulate V, as is the hard direction (I-29) of the Z-theorem.