MATH 2260H - Propositions I-17 to I-22

2021-01-25 ①

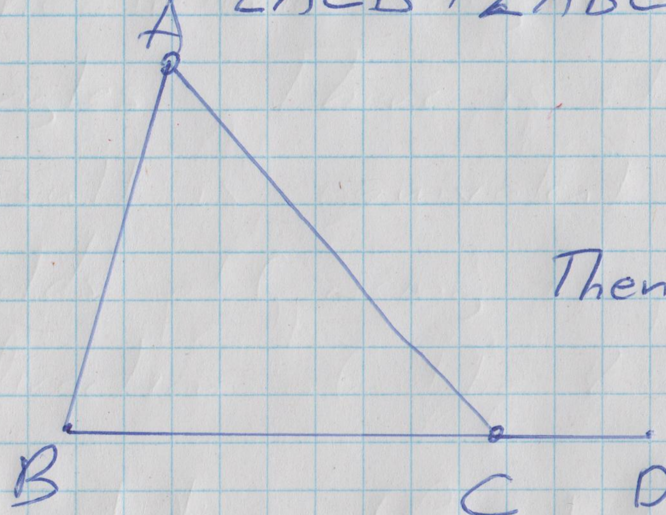
or, more about triangles

We are headed for showing that the sum of the interior angles of a triangle is two right angles. There is still quite a slog ahead - the result is equivalent to Postulate V and Euclid holds off using it until Prop. I-29 (the hard direction of the Z-theorem).

Last time: (Prop. I-16) If any side of a triangle is extended, the resulting external angle is greater than either of the internal and opposite angles.

Prop. I-17: The sum of any two internal angles of a triangle is less than two right angles.

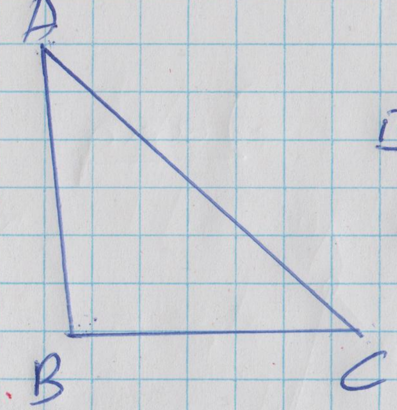
proof: Given triangle $\triangle ABC$, let's show that
$\angle ACB + \angle ABC < 2\llcorner = \frown$.



Extend BC past C to D.
By Prop I-16, $\angle ABC < \angle ACD$.
Then $\angle ABC + \angle ACB < \angle ACD + \angle ACB = \angle BCD = \frown$ //

For any other combination of two of the ~~other~~ internal angles, extend a different and give a similar argument.

Prop. I-18: In any triangle, the greater side subtends the greater angle.



ie $|AC| > |AB| \Rightarrow \angle ABC > \angle ACB$

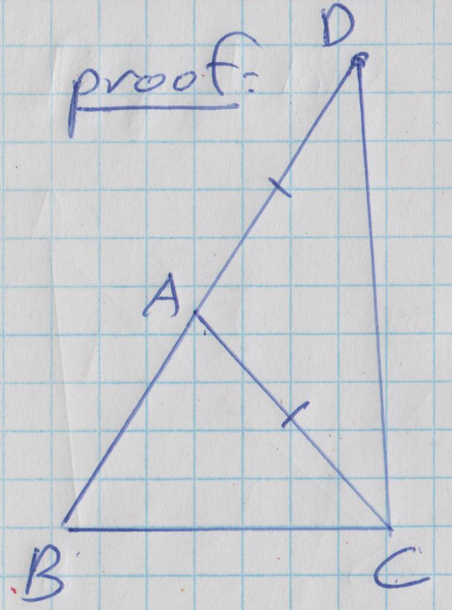
proof: Left to you to read in the *Elements*. //

Prop. I-19: In any triangle, the greater angle subtends the greater side.

proof: Left to you to read in the *Elements*. //

Proposition I-20: In any triangle, any two sides have a combined length greater than the remaining side.

<u>proof:</u>



Suppose we are given ΔABC.

We will try to show that $|AB| + |AC| > |BC|$.

Extend BA past A to a point D such that $|AD| = |AC|$. Connect C to D.

Since $|AD| = |AC|$, $\angle ADC = \angle ACD$ (Prop. I-5).

So $\angle BCD > \angle ACD = \angle ADC$, and hence

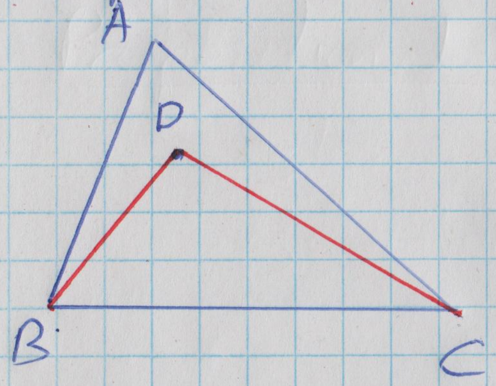
$\angle BDC =$

in ΔBCD, $|BD| > |BC|$ (by Prop. I-19).

But $|BD| = |AB| + |AD| = |AB| + |AC|$,

so $|AB| + |AC| > |BC|$. //

<u>Prop. 21:</u>



Suppose D is a point in the interior of ΔABC.

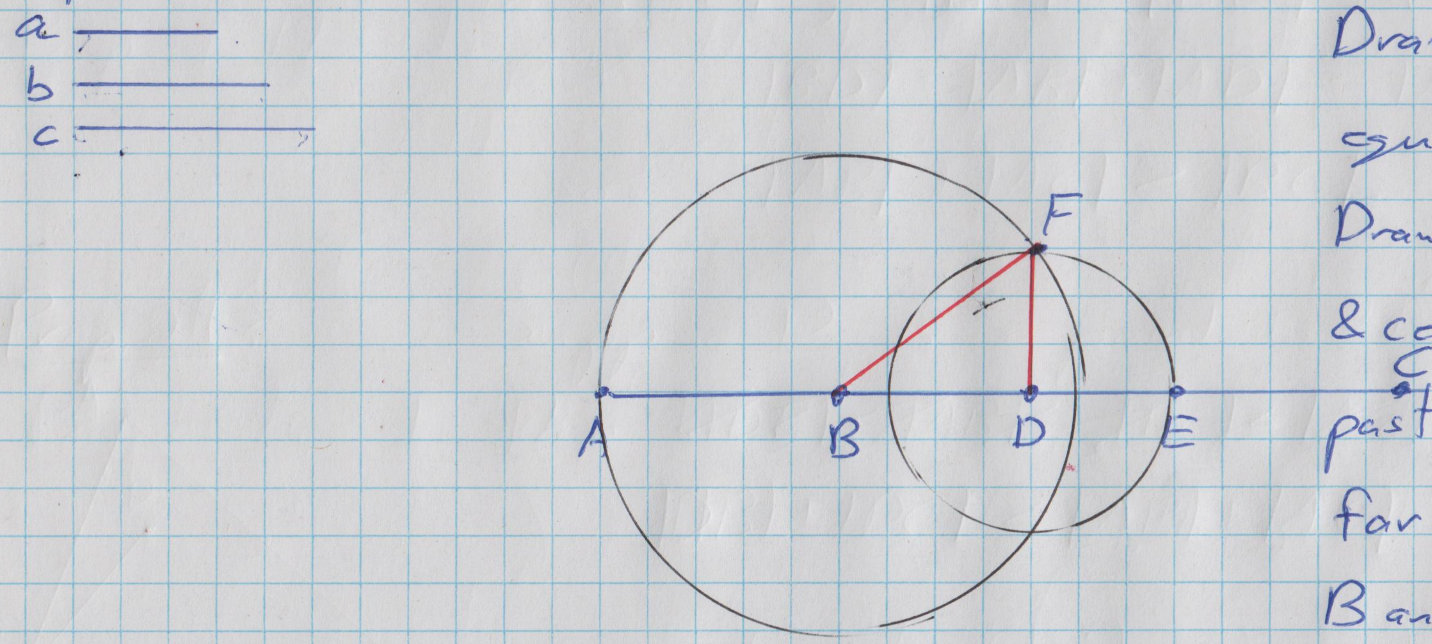
Then $\cancel{|DB| < |AB|}$, $\cancel{|DC| < |AC|}$, and $\angle BAC < \angle DAC$.

$|DB| + |DC| < |AB| + |AC|$

<u>proof:</u> Left to <u>you</u> to read in the <u>Elements</u>. //

<u>Prop. I-22</u>: Given three line segments a, b, c such that the sum of the lengths of any two are greater than the length of the third, you can construct a triangle with sides equal to the lengths of the given line segments.

<u>proof</u>: Given the line segments, pick a point A somewhere.



Draw a line segment AB equal in length to C. [I-3] Draw a circle of radius AB & centre ~~A~~B. Extend AB past B & the circle to C far away. Find D between B and C s.t. $|BD| = |b|$.

~~Draw a~~ Find a point E on DC ~~s~~ such that $|DE| = |a|$. So E is outside the previous circle. Draw the circle with centre D and radius DE. This intersects the previous circle at F. Then $\triangle BDF$ is the required triangle. //