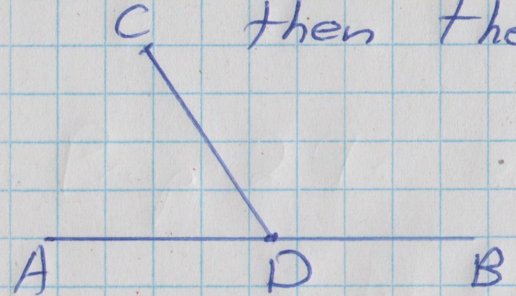
Math 2260H - Angles & Triangles

or, Propositions I-13 to I-16

Prop. I-13: "If a straight line be stood on another straight line, then the angles it makes add up to two right angles."

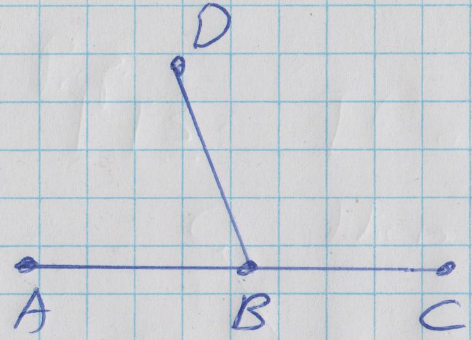


ie Given two line segments AB and CD such that D is between A and B on AB,

$$\angle ADC + \angle CDB = 2\llcorner = \frown .$$

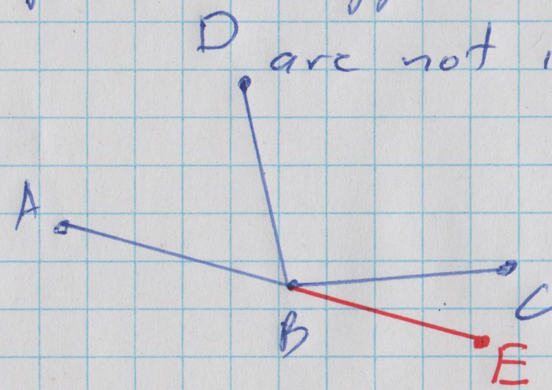
proof: $\angle ADC + \angle CDB = \angle ADB = \frown = 2\llcorner$. //

Prop. I-14: "If two straight lines, not lying on the same side, make adjacent angles equal to two right angles at some point, then the two straight lines will be straight on with respect to one another."



ie If AB and BC are line segments and BD is also a line segment such that $\angle ABD + \angle DBC = \frown = 2\llcorner$, then A, B, C are "collinear" [all on the same straight line].

proof: Suppose, by way of contradiction that A, B, and C are not in a straight line (ie $\angle ABC \neq \frown$).



Extend AB past B to a point E.

Two cases: (i) E is on the same side of BC as D is. [see the Elements]

(ii) E is on the other side of BC from D. [we'll do this one] case (ii): $\angle ABE = \underbrace{\angle ABD + \angle DBC} + \angle CBE$

$$\overset{''}{\underline{\frown}} = \underline{\frown} + \angle CBE$$

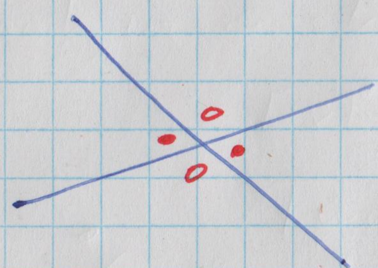
Thus $\underline{\frown} = \underline{\frown} + \angle CBE > \underline{\frown}$, which is a contradiction. [A similar argument in case (i) - with the roles of C & E reversed.]

Thus,

the assumption that A, B, & C are not collinear must be wrong.

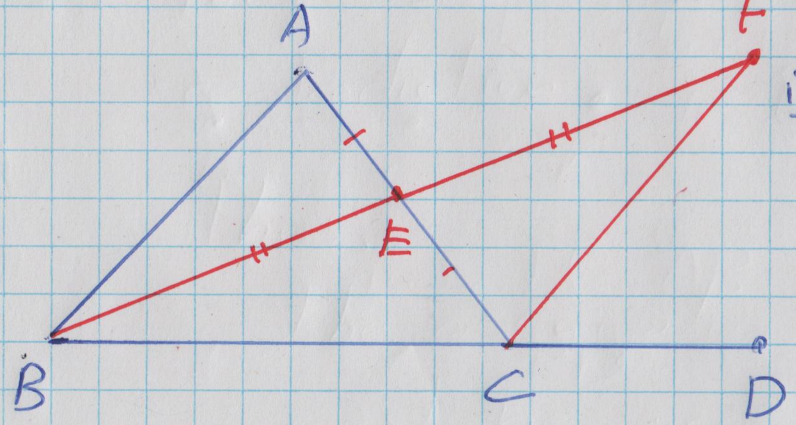
$\therefore$ A, B, C are collinear. //

<u>Prop:</u> I-15 (Opposite Angles Theorem)



We did this several lectures ago using a different proof from Euclid's.

Prop I-16: "For any triangle, when one of the sides is extended, the external angle is greater than each of the internal and opposite angles."



ie Given ΔABC and a point D on an extension of BC past C,
$\angle ACD > \angle ABC$
and $\angle ACD > \angle BAC$.

proof: Let E be the midpoint of AC (ie $|AE| = |EC|$ & E is on AC).
Connect B to E and extend BE past E to a point F such that $|EF| = |BE|$, and the connect ~~E~~ F to C.
By the Opposite Angle Theorem (Prop. I-15), $\angle AEB = \angle CEF$. Since we also have $|AE| = |CE|$ and $|BE| = |FE|$, it follows by the S-A-S congruence criterion (Prop. I-8) that $\Delta AEB \cong \Delta CEF$.

It follows immediatly that

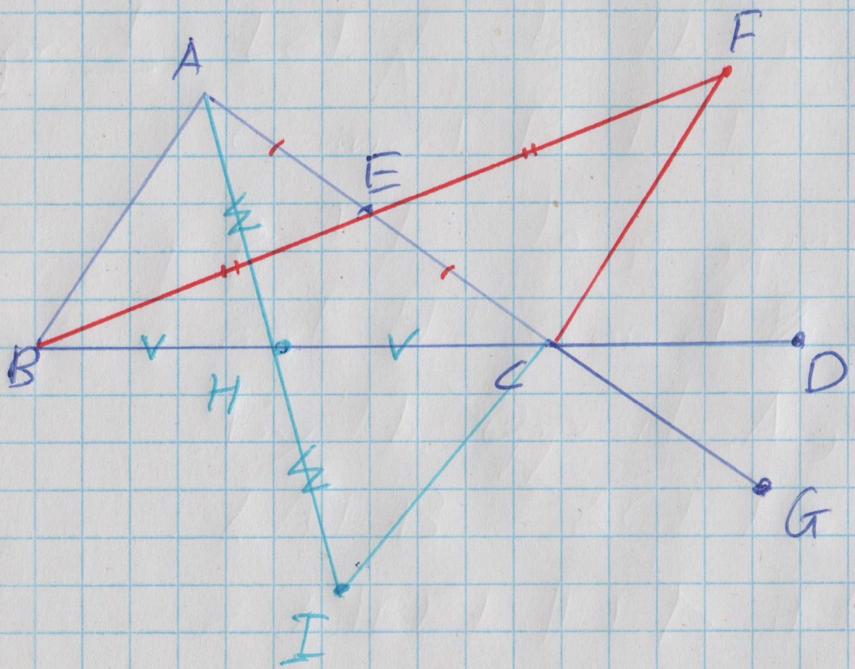
$$\angle BAE = \angle FCE,$$

so $\angle ACD = \angle DCF + \angle FCE$

$\qquad\qquad > \angle FCE = \angle BAE = \angle BAC$

ie $\angle ACD > \angle BAC$.

We still have to show that $\angle ACD > \angle ABC$



Extend AC past C to G.
Let H be the midpoint of BC
Connect A to H & extend
AH past H to I s.t. $|AH| = |HI|$
& then connect I to C.
We can now replicate the previous
argument with H playing E, G playing D,
I playing F and so on, to get $\angle ACD = \angle BCG > \angle ABC$. //