MATH 2260H - Euclidean Geometry

2021-01-12 ①

For those who find Fitzpatrick's translation of Euclid a bit sparse and "old-fashioned", there are various modernized versions out there. Two good ones are:

1) Geometry from Euclid to Knots, by Saul Stahl.
   Currently in print from Dover Publications. (This is the book I adopted Postulates S and A from.)

2) Euclid's Elements Redux, by Daniel Callahan*.
   Released under a CC-ASA‡ license. You can buy print copies from Amazon or download an electronic copy for free from various sites, such as archive.org .

* building on work by Casey & Heath, and Heiberg before them.

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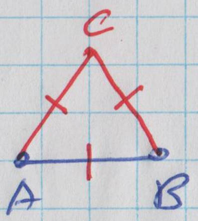
Book of the Elements

Prop. in that Book

Recall: We proved Proposition I-1 of the *Elements*:



Given a line-segment, we can construct an equilateral triangle which has that line-segment as one of its sides.

We'll use Prop. I-1 to help prove:

Prop. I-2: "To place a straight-line equal to a given straight-line at a given point."

ie Given a point A and a line-segment BC, find a point D such that $|AD| = |BC|$.

proof: This naturally has 3 cases, of increasing difficulty:

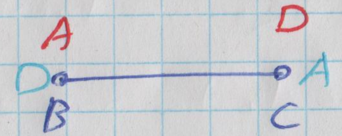
(1) A is an endpoint of BC (ie $A=B$ or $A=C$)

(2) A is on BC between B and C.

(3) A is not on BC.

Euclid only proves case (3). (In general, he only proves the hardest cases and leaves the others for the reader.)

We'll prove cases (1) & (2). [(2) uses similar ideas to the proof of (3).]

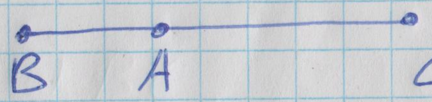


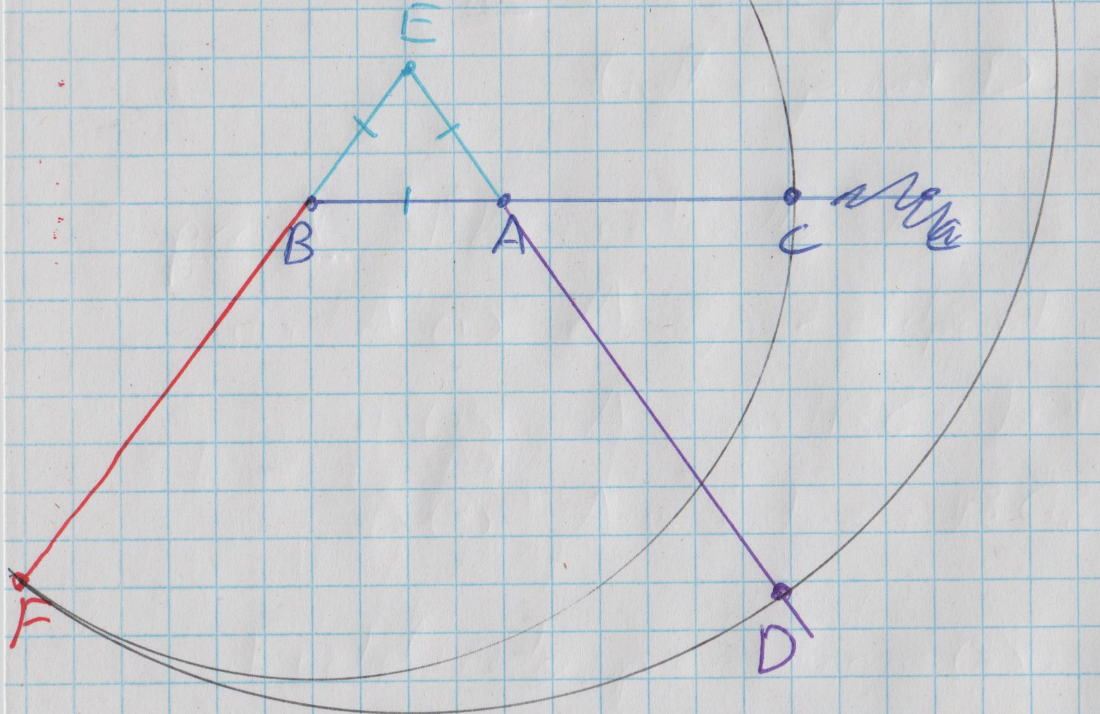
(1) $A = B$ or $A = C$.

Let $D = C$ if $A = B$, and let $D = B$ if $A = C$.

Either way, $|AD| = |BC|$, since they are the same line segment.

(2) A is a point on BC between B and C.





i) Use I-1 to construct $\triangle BAE$ which is equilateral

ii) Draw the circle with centre B & radius BC (Postulate III)

iii) Extend EB past B until it meets the circle in ii), (Postulates II & 5) at F.

iv) Draw the circle with centre E & radius EF. (Post. III)

v) Extend EA past A until it meets the circle in iv) at D. (Post. II & 5)

We need to verify that $|AD| = |BC|$.

$$|ED| = |EF| \quad \text{(both are radii of the circle in iv)}$$

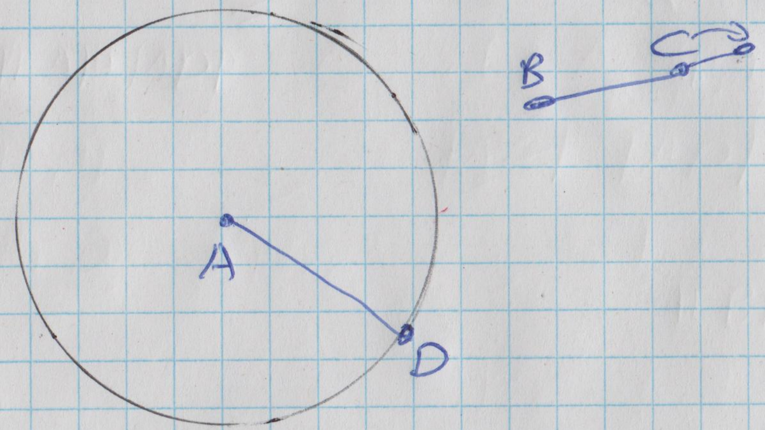
$$\begin{array}{ccl} = & & \shortparallel \\ |EA| + |AD| & & |EB| + |BF| \quad \text{(decomposing each line segment into pieces)} \\ & & \shortparallel \\ & & |EB| + |BC| \quad \text{(since } BC \text{ \& } BF \text{ are both radii of the circle in iii)} \end{array}$$

Since $|EA| = |EB|$ (both are sides of the equilateral ΔABE), it follows that $|AD| = |BC|$.

(3) A is not on BC – check out Euclid's proof, which uses some of the same ideas as case (2). //

Corollary: Given a point A and a line-segment BC, we can draw a circle with centre A and radius equal to $|BC|$.

proof:



Find the D s.t. $|AD| = |BC|$ (by I-2) and draw the circle with radius AD and centre A. (Post. III).

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This means that even though Post III only allows drawing circles with the radius attached to the centre, we can actually draw circles with a given centre and a radius given by a non-attached line-segment.

Next time: Side-Angle-Side (SAS) congruence criterion for triangles.