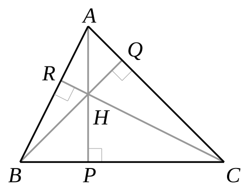
Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2014

Take-Home Final Examination
Due on Friday, 18 April, 2014.

Instructions: Do both of parts △ and □, and, if you wish, part ◯ as well. Show all your work. You may use your textbooks and notes, as well as any handouts and returned work, from this and any other courses you have taken or are taking now. You may also ask the instructor to clarify the statement of any problem, and use calculators or computer software to do numerical computations and to check your algebra. However, *you may not consult any other sources, nor consult or work with any other person on this exam.*

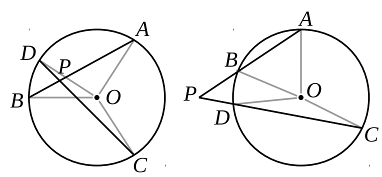
Part △. Do *all* of problems **1** – **4**. *[40 = 4 × 10 each]*

1. Suppose H is the orthocentre of $\triangle ABC$. Show that C is the orthocentre of $\triangle ABH$.

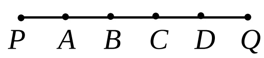


2. Suppose the chords AB and CD of a circle with centre O intersect at a point P that is not on the circle. Show that:

a. $\angle APC = \frac{1}{2}(\angle AOC + \angle BOD)$ if P is inside the circle. *[5]*
b. $\angle APC = \frac{1}{2}(\angle AOC - \angle BOD)$ if P is outside the circle. *[5]*



3. Suppose a line segment PQ is given. Give a detailed construction using Euclid's system, as augmented in the textbook, of points A, B, C, and D between P and Q such that $|PA| = |AB| = |BC| = |CD| = |DQ|$.



4. Consider the following axioms:

I. Any two points are on an unique line.
II. Any two lines have an unique point in common.
III. There are four points such that no three are on the same line.

Find as many essentially different examples of configurations of points and lines that satisfy axioms **I** and **II**, but do not satisfy axiom **III**, as you can.

[The axioms given above are those for a *projective plane*; configurations satisfying **I** and **II**, but not **III**, are called *degenerate planes*.]

Part □. Do any *four* (4) of problems **5 – 10**. *[40 = 4 × 10 each]*
Please draw the relevant pictures in each case!

5. Suppose O is the circumcentre of $\triangle ABC$ and points X, Y, and Z are chosen so that BC is the perpendicular bisector of OX, AC is the perpendicular bisector of OY, and AB is the perpendicular bisector of OZ. Show that $\triangle XYZ \cong \triangle ABC$.

6. A chord PQ of a circle is tangent to a smaller circle with the same centre. Assuming that $|PQ| = 4\ cm$, find the area of the region between the two circles.

7. Suppose $\triangle ABC$ has a right angle at C and let CD be the altitude from C of this triangle. Let r, q, and p be the radii of the incircles of $\triangle ABC$, $\triangle CAD$, and $\triangle CBD$, respectively. Show that $r + q + p = |CD|$

8. Suppose A, B, C, D, E, and F are points on a circle, arranged clockwise. Let U be the intersection of AE and BF, V be the intersection of AD and CF, and W be the intersection of BD and CE. Show that U, V, and W are collinear.

9. Join each vertex of $\triangle ABC$ to the points dividing the opposite side into equal thirds and let X, Y, and Z be the points of intersection of the pairs of these lines closest to BC, AC, and AB, respectively. Show that the sides $\triangle XYZ$ are parallel to corresponding sides of $\triangle ABC$ and that $\triangle XYZ \sim \triangle ABC$.

10. Suppose $ABCD$ is a cyclic quadrilateral, *i.e.* A, B, C, and D are points on a circle, given in order going around the circle. Show that if we join each of A, B, C, and D to the orthocentre of the triangle formed by the other three, then the resulting line segments all intersect in a common midpoint M.

[Total = 80]

Part ○. Bonus!

0. Write an original poem about geometry. *[1]*

00. Give an example of two triangles $\triangle ABC$ and $\triangle DEF$ which are *not* congruent, but which nevertheless have the same centroid G, orthocentre H, incentre I, and circumcentre O. *[1]*

I HOPE THAT YOU ENJOYED THIS COURSE!
HAVE A GREAT SUMMER!