

Mathematics 2260H – Geometry I: Euclidean geometry

TRENT UNIVERSITY, Winter 2014

Assignment #3

Circles for triangles

Due on Friday, 14 February, 2014.

The circle that passes through all three vertices of $\triangle ABC$ is called the *circumcircle* of the triangle and its centre is the triangle's *circumcentre*.

1. Show that the perpendicular bisectors of the sides of $\triangle ABC$ meet in a point that is equidistant from all three vertices. [3]
2. Use 1 to help show that any triangle has a unique circumcircle. [2]
3. Suppose P is a point outside of the circle centred at A , and B and C are points on the circle such that PC and PB are both tangent to the circle. Show that PA bisects $\angle BPC$. [2]

NOTE: You may use the fact that if PB is tangent at B to a circle centred at A , then PB is perpendicular to the radius AB . (And similarly for C , of course!)

4. Show that there is a unique circle with its centre inside $\triangle ABC$ that is tangent to all three sides of the triangle. [3]

The centre of the circle in 4 is called the *incentre* of the triangle; the circle itself is called the *incircle* of the triangle.