Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2014

Assignment #2
Polygons, congruence, and similarity
Due on Friday, 31 January, 2014.

A polygon is a figure $A_1A_2\dots A_n$ consisting of points, called *vertices* [each one is a *vertex*], A_1, A_2, ... , A_n, and the line segments, called *sides*, A_1A_2, A_2A_3, ... , $A_{n-1}A_n$, A_nA_1. We will usually assume that the vertices are all distinct and the sides do not intersect except as required by the definition above. A polygon with n vertices is often generically referred to as an *n-gon*. 3-gons are usually called *triangles*, 4-gons *quadrilaterals*, 5-gons *pentagons*, 6-gons *hexagons*, and so on. A polygon is *regular* if all the sides are the same length and the interior angles between the sides meeting at each vertex are all the same, too. In particular, a triangle is regular if it is an equilateral triangle, and a quadrilateral is regular if it is a square.

Two polygons are *congruent* if they are identical except for their placement in the plane. That is, $A_1A_2\dots A_n$ is congruent to $B_1B_2\dots B_n$, often written as $A_1A_2\dots A_n \cong B_1B_2\dots B_n$, if corresponding sides and interior angles are all equal, *i.e.* $|A_1A_2| = |B_1B_2|$, $|A_2A_3| = |B_2B_3|$, ... , $|A_nA_1| = |B_nB_1|$, and $\angle A_1A_2A_3 = \angle B_1B_2B_3$, ... , $\angle A_nA_1A_2 = \angle B_nB_1B_2$.

Two polygons are *similar* if they are identical except for their size and placement in the plane. That is, $A_1A_2\dots A_n$ is similar to $B_1B_2\dots B_n$, often written as $A_1A_2\dots A_n \sim B_1B_2\dots B_n$, if corresponding interior angles are all equal, *i.e.* $\angle A_1A_2A_3 = \angle B_1B_2B_3$, ... , $\angle A_nA_1A_2 = \angle B_nB_1B_2$, and the ratios of corresponding sides are all the same, *i.e.* $\frac{|A_1A_2|}{|B_1B_2|} = \frac{|A_2A_3|}{|B_2B_3|} = \cdots = \frac{|A_nA_1|}{|B_nB_1|}$.

We will mainly be concerned with triangles when dealing with congruence and similarity, but we will sometimes deal with other polygons, and may even extend the notions to figures other than polygons on occasion.

1. Show that congruence implies similarity in the Euclidean plane, *i.e.*
$$A_1A_2\dots A_n \cong B_1B_2\dots B_n \Longrightarrow A_1A_2\dots A_n \sim B_1B_2\dots B_n,$$
but not the other way around. *[2]*

Since we haven't yet developed all the Euclidean tools needed, you may, if you wish, use trigonometry and the fact that the interior angles of a triangle sum to two right angles to help do the following problems.

2. Prove the Angle-Side-Angle (ASA) congruence criterion for triangles, *i.e.* if $\angle ABC = \angle DEF$, $|BC| = |EF|$, and $\angle BCA = \angle EFD$, then $\triangle ABC \cong \triangle DEF$. *[2]*

3. Prove the Angle-Angle (AA) similarity criterion for triangles, *i.e.* if $\angle ABC = \angle DEF$ and $\angle BCA = \angle EFD$, then $\triangle ABC \sim \triangle DEF$. *[2]*

4. Prove the Side-Angle-Side (SAS) similarity criterion for triangles, *i.e.* if $\angle ABC = \angle DEF$ and $\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|}$, then $\triangle ABC \sim \triangle DEF$. *[2]*

5. Suppose P and Q are the midpoints of sides AB and AC in $\triangle ABC$. Show that $\triangle ABC \sim \triangle APQ$ and $|BC| = 2|PQ|$. *[2]*