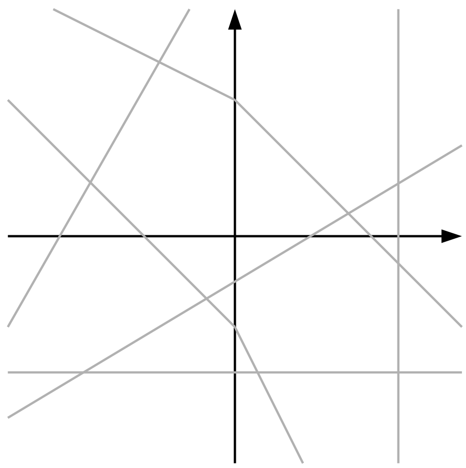
Mathematics 2260H – Geometry I: Euclidean geometry
Trent University, Winter 2014

Assignment #1
The Moulton plane
Due on Friday, 17 January, 2014.

The *Moulton plane* is an example of a plane geometry that is not the familiar Euclidean plane. One way to define it is to start with the usual Cartesian coordinate plane and redefine the lines of slope m by redefining the operation of multiplication on the real numbers. The new operation of multiplication, which we will denote by $\star$, is defined in terms of the usual operatiuon of multiplication as follows:

$$u \star v = \begin{cases} uv & \text{if } u \geq 0 \text{ or } v \geq 0 \text{ (or both)} \\ \frac{1}{2}uv & \text{if } u \leq 0 \text{ and } v \leq 0 \end{cases}$$

The points of the Moulton plane are just the points (x, y), for x, $y \in \mathbb{R}$, of the Cartesian plane. The lines of the Moulton plane include the vertical lines, $x = a$ for $a \in \mathbb{R}$, of the Cartesian plane, plus all the "lines" satisfying the equation $y = m \star x + b$, where m, $b \in \mathbb{R}$. In practice, this means that all lines which are vertical, or horizontal, or have positive slope in the Cartesian plane are still lines of the Moulton plane. However, lines of negative slope are bent to make them only half as steep to the left of the y-axis:



1. Determine as fully you can which of Euclid's five Postulates are satisfied in the Moulton plane. *[6]*

2. Show that the Moulton plane does not satisfy Desargue's Theorem. [Look it up! We will see later that the Euclidean does satisfy this theorem.] *[4]*

References

1. *A Simple Non-Desarguesian Plane Geometry*, Forest Ray Moulton, *Transactions of the American Mathematical Society* **3** (1902), pp. 192–195.