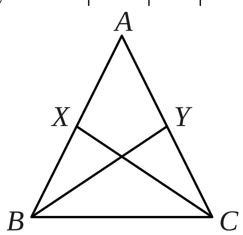
Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2013

Take-Home Final Examination
Due on Friday, 19 April, 2013.

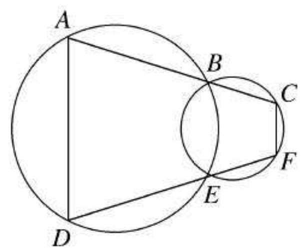
Instructions: Do both of parts $\bigtriangledown$ and $\square$, and, if you wish, part $\odot$ as well. Show all your work. You may use any sources you like, provided that you give all that contributed to your final solutions due credit, and that you adapt whatever arguments you find to work on the basis of the material covered in this course. You may also ask the instructor to clarify any of the problems, but *you may not consult or work with any other person.*

Part $\bigtriangledown$. Do *all* of problems **1** – **4**. *[40 = 4 × 10 each]*

1. In $\triangle ABC$, $|AB| = |AC|$ and X and Y are points on AB and AC, respectively, such that $|AX| = |AY|$. Show, in detail, that $|BY| = |CX|$.

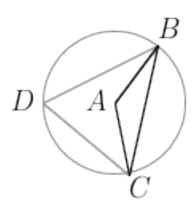


2. Two circles intersect at points B and E. A and D on the first circle, and C and F on the second circle, are points such that A, B, C are collinear and D, E, F are collinear. Show that $AD \parallel CF$.



3. Suppose Suppose the centroid of $\triangle ABC$ is also its incentre. Prove that $\triangle ABC$ must be equilateral or give an example showing it doesn't have to be equilateral.

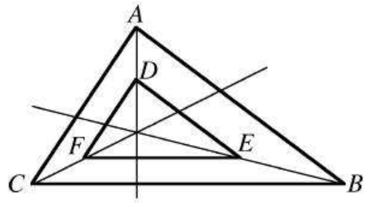
4. Suppose $\triangle ABC$ is isosceles, with $|AB| = |AC|$, and D is a point on the same side of BC as A such that $\angle BAC = 2\angle DBC$. Show that A is the circumcentre of $\triangle DBC$.



Part $\square$. Do any *four* (4) of problems **5** – **11**. *[40 = 4 × 10 each]*

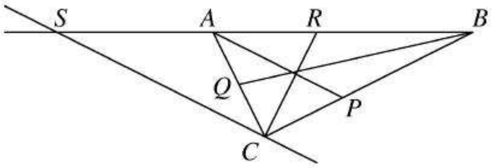
5. Given a square $\square ABCD$, use Euclid's system to show that there is an equilateral triangle $\triangle DEF$ with the same area as $\square ABCD$.

6. Suppose $\triangle DEF$ lies inside $\triangle ABC$, with $AB \parallel DE$, $AC \parallel DF$, and $BC \parallel EF$. Show that the lines AD, BE, and CF are concurrent.



7. Suppose the radius of the incircle of $\triangle ABC$ is r and the *semiperimeter* of the triangle is $s = \frac{1}{2}(|AB| + |BC| + |CA|)$. Show that the area of the triangle is equal to rs.

8. Suppose AP, BQ, and CR are the angle bisectors of $\triangle ABC$, and suppose that S is a point on the line AB such that CS is perpendicular to CR. Show that P, Q, and S are collinear.



9. Suppose a geometry satisfies the following axioms:

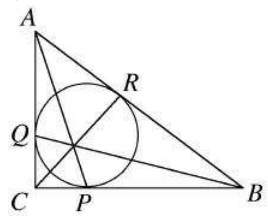
I. Any two points are on an unique line.

II. Any two lines have an unique point in common.

III. There are four points such that no three are on the same line,

Show that the geometry must have at least seven points and at least seven lines.

10. Suppose that the incircle of $\triangle ABC$ is tangent to the sides BC, AC, and AB at the points P, Q, and R, respectively. Show that AP, BQ, and CR are concurrent.



11. Suppose $ABCD$ is a cyclic quadrilateral, *i.e.* A, B, C, and D are points on a circle, given in order going around the circle. Show that if we join each of A, B, C, and D to the orthocentre of the triangle formed by the other three, then the resulting line segments all intersect in a common midpoint M.

Part $\odot$.

0. Write an original poem about geometry. *[2]*

[Total = 80]

I HOPE THAT YOU ENJOYED THIS COURSE!
HAVE A GOOD SUMMER!