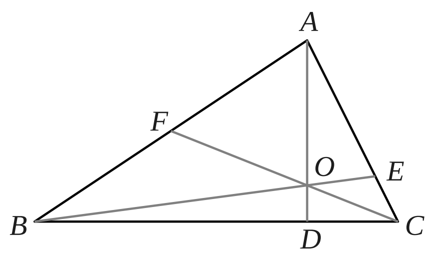
Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2013

Assignment #7
Ceva's Theorem
Due on Friday, 8 March, 2013.

The following result was first proved by the medieval Arab mathematician Yusuf al-Mu'taman ibn Hūd, who was also ruler of Zaragoza from 1082 to 1085. His work was lost for many centuries, though, and in the meantime it was rediscovered and proved again by Giovanni Ceva (1647-1734), an Italian Jesuit mathematician.

CEVA'S THEOREM. Suppose D, E, and F are points on the sides BC, AC, and AB, respectively, of $\triangle ABC$. Then AD, BE, and CF all meet in a single point O if and only if $\frac{|AF|}{|FB|} \cdot \frac{|BD|}{|DC|} \cdot \frac{|CE|}{|EA|} = 1$.



The statement above is not quite the most general: with some additional definitions, there is a version that also works when two of D, E, and F are on extensions of the sides and O is outside the triangle. However, the limited version given above is still useful.

1. Prove half of Ceva's Theorem: Suppose D, E, and F are points on the sides BC, AC, and AB, respectively, of $\triangle ABC$. Then if AD, BE, and CF all meet in a single point O, then $\frac{|AF|}{|FB|} \cdot \frac{|BD|}{|DC|} \cdot \frac{|CE|}{|EA|} = 1$. *[10]*

Hint: Rewrite $\frac{|AF|}{|FB|} \cdot \frac{|BD|}{|DC|} \cdot \frac{|CE|}{|EA|}$ in terms of the areas of certain triangles in the picture.