

Mathematics 2260H – Geometry I: Euclidean geometry

TRENT UNIVERSITY, Winter 2013

Assignment #4

Congruence and Similarity

Due on Friday, 8 February, 2013.

1. Show that if $\angle ABC = \angle DEF$, $\angle BCA = \angle EFD$, and $|CA| = |FD|$, then $\triangle ABC \cong \triangle DEF$. [5]

NOTE: That is, show that the Angle-Angle-Side (AAS) congruence criterion for triangles works.

DEFINITION. $\triangle ABC$ and $\triangle DEF$ are *similar*, often denoted by $\triangle ABC \sim \triangle DEF$, if the ratios of (the lengths of) corresponding sides are all the same, *i.e.* if $\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$.

2. Show that congruence implies similarity for triangles, *i.e.* $\triangle ABC \cong \triangle DEF$ implies that $\triangle ABC \sim \triangle DEF$. Give an example to show that the converse is not necessarily true. [1]
3. Use trigonometry to show that if $\angle ABC = \angle DEF$ and $\angle BCA = \angle EFD$, then $\triangle ABC \sim \triangle DEF$.

NOTE: That is, show that the Angle-Angle (AA) criterion for similarity works. Besides trigonometry, you may assume – as trigonometry in Euclidean space does – that the sum of the interior angles of a triangle is $180^\circ = \pi \text{ rad} =$ a straight angle.

We will revisit the definition and basics of similarity once we start using the parallel axiom (which similarity really needs), but introducing the concept now will allow us to investigate some uses of it before that.