Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2013

Assignment #1
A geometry on a cylinder
Due on Friday, 18 January, 2013.

We will define a geometry $\mathcal{G}$ of points and *lines* on (the surface of) the cylinder $x^2 + y^2 = 1$.

The points of the geometry $\mathcal{G}$ are the points of the cylinder, *i.e.* the points (x, y, z) in three-dimensional Cartesian coordinates which satisfy the equation $x^2 + y^2 = 1$. The *lines* of the geometry $\mathcal{G}$ are the curves on the cylinder obtained by intersecting a plane through the origin in three-dimensional space with the cylinder. (Note that every plane through the origin does indeed intersect the cylinder.) Your task will be to determine some of the basic properties of $\mathcal{G}$.

1. Suppose P and Q are two distinct points of $\mathcal{G}$. Show that there is at least one *line* ℓ of $\mathcal{G}$ which passes through both P and Q. Is such a line necessarily unique? Explain why or why not. *[3]*

2. Give (different! :-) examples to show that it is possible for distinct *lines* ℓ and m of $\mathcal{G}$ to intersect in 0 or 2 points of $\mathcal{G}$. *[3]*

3. Explain why distinct *lines* ℓ and m of $\mathcal{G}$ cannot intersect in just 1 point of $\mathcal{G}$. *[1]*

4. Determine whether or not *Playfair's Axiom*,

> If P is a point not on the *line* ℓ, then there is a unique *line* m through P such that l and m do not intersect in any point.

is true in $\mathcal{G}$. *[3]*

Bonus. Suppose a right-angled triangle on the surface of a sphere of radius R has sides of lengths a and b and a hypotenuse of length c respectively. (Note that the sides of the triangle are all pieces of great circles of the sphere.) Show that $\cos(a/R) \cdot \cos(b/R) = \cos(c/R)$. *[1]*

NOTE: I learned this formula from a short story, *The New Physics: The Speed of Lightness, Curved Space, and Other Heresies*, by Charles Sheffield. It first appeared in ANALOG, Vol. 100, No. 9, September 1980, and was reprinted in the anthology *Hidden Variables,* by Charles Sheffield, Ace Science Fiction, New York, 1981. It's a wonderful spoof of Galileo, Einstein, and physics in general. (All the formulas in it are actually correct in context, I think ... :-)