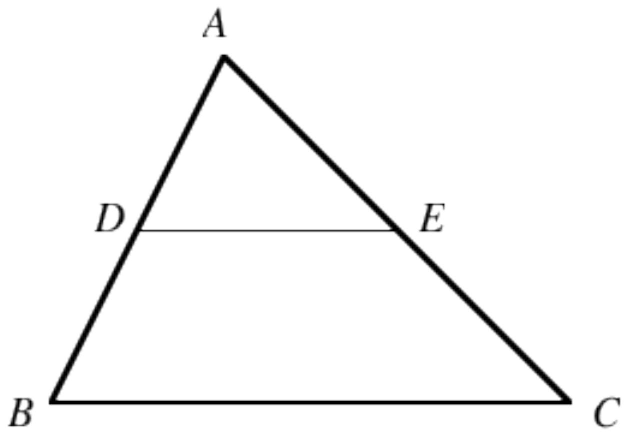
Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2012

Solutions to Assignment #5
Tinkering with triangles

In both of the questions below suppose D and E are the midpoints of sides AB and AC, respectively, of $\triangle ABC$.



1. Show that $DE \parallel BC$ and $BC = 2DE$. *[5]*

HINT: First show that $\triangle ABC \sim \triangle ADE$.

SOLUTION. As D and E are the midpoints of AB and AC, respectively, $\frac{AD}{AB} = \frac{1}{2} = \frac{AE}{AC}$. Since $\angle DAE = \angle BAC$ (they're the same angle, after all), it follows by the side-angle-side similarity criterion (Assignment #4, Question **3**) that $\triangle ADE \sim \triangle ABC$. Hence $\frac{DE}{BC} = \frac{AD}{AB} = \frac{1}{2}$, so $BC = 2DE$.

To see that $DE \parallel BC$, observe that because $\triangle ADE \sim \triangle ABC$, we have $\angle ADE = \angle ABC$. By one of the many close relatives of the Z-Theorem (see Euclid's Proposition I-29, *i.e.* Theorem 3.1.1 in our text, for a succinct statement of the basic ones in a single package), it follows that $DE \parallel BC$, as desired. ∎

2. Show that $\triangle ABC$ has four times the area of $\triangle ADE$. *[5]*

HINT: Show that $\triangle ABC$ can be divided up into four triangles, each of which is congruent to $\triangle ADE$.

SOLUTION. Let F be the midpoint of BC. Then, using arguments similar to those in the solution to question **1** above, we get that $AC = 2DF$ and $AB = 2EF$. Since D, E, and F are the midpoints of AB, AC, and BC respectively, it follows that $AD = DB = FE = EF$, $AE = DF = FD = EC$, and $DE = BF = ED = FC$. It follows by the side-side-side congruence criterion that $\triangle ADE \cong \triangle DBF \cong \triangle EFD \cong \triangle EFC$. Since these four triangles, each congruent to $\triangle ADE$, put together make up $\triangle ABC$, their combined area is equal to the area of $\triangle ABC$. Thus $\triangle ABC$ has four times the area of $\triangle ADE$. ∎