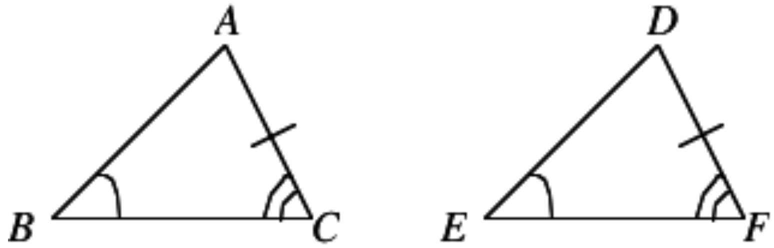
Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2012

Solution to Assignment #3
Angle-Angle-Side!

1. Show that the Angle-Angle-Side congruence criterion actually works. That is, show that if $\angle ABC = \angle DEF$, $\angle BCA = \angle EFD$, and $CA = FD$, then $\triangle ABC \cong \triangle DEF$. *[10]*



SOLUTION. Apply $\triangle ABC$ to $\triangle DEF$ so that A is on D, AC is along DF, and B and E are on the same side of DF. Since $AC = DF$, it follows that C is on F as well. Since $\angle BCA = \angle EFD$, it then follows that BC is along EF.

We claim that B must be on E as well. Suppose, by way of contradiction that B falls strictly between E and F. Then $\angle DEB + \angle DBE = \angle DEF + \angle DBE = \angle ABC + \angle ABE$ amounts to a straight angle, contradicting the fact (Euclid's Propositiom I-17) that any two internal angles of a triangle sum to less than a straight angle. A similar argument by contradiction show that E cannot fall strictly between B and F either. It follows that B must be on E.

Since the triangles coincide when one is applied to the other, they must be congruent. (Feel free to use your favourite congruence criterion, or just satisfy the definition of congruence ...) ■