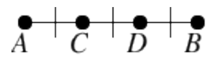
Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2012

Take-Home Final Examination
Due on Friday, 20 April, 2012.

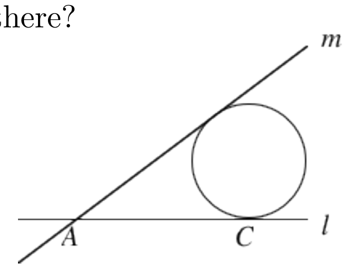
Instructions: Do both of parts △ and □, and, if you wish, part ◯ as well. Show all your work. You may use any sources you like, provided that you give all that contributed to your final solutions due credit, and that you adapt whatever arguments you find to work on the basis of the material covered in this course. You may also ask the instructor to clarify any of the problems, but *you may not consult or work with any other person.*

Part △. Do *all* of problems **1 – 4**. *[40 = 4 × 10 each]*

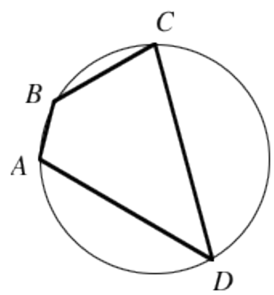
1. Suppose a line segment AB is given. Use Euclid's system, as augmented in the textbook, to construct points C and D on AB such that C is between A and D, D is between C and B, and $AC = CD = DB$.



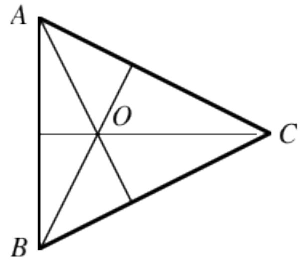
2. Suppose m and l are lines intersecting at point A and C is another point on l. Show that there is a circle which is tangent to l at C which is also tangent to the line m. How many such circles are there?



3. Suppose $ABCD$ is a convex quadrilateral. Show that it is *cyclic* (*i.e.* there is a circle passing through all four vertices) if and only if $\angle ABC + \angle ADC$ is a straight angle.



4. Suppose O is the orthocentre of $\triangle ABC$. Show that A is the orthocentre of $\triangle OBC$.



Part □. Do any *four* (4) of problems **5 – 11**. *[40 = 4 × 10 each]*

5. Show that a point P is equidistant from points A and B if and only if P is on the perpendicular bisector of AB.

6. Suppose A and B are points on a circle with centre C and P is a point where the tangents to the circle at A and B intersect. Show that $\angle APC = \angle BPC$.

7. Suppose a line segment AB and a point C are given. Show how to construct a line segment CD such that $AB = CD$ and $AB \parallel CD$.

8. Show that the centre of the nine-point circle of a triangle is on the Euler line of the triangle. Where on the line is it in relation to the triangle's centroid, circumcentre, and orthocentre?

9. Prove that if a triangle has sides of lengths a, b, and c, then its area is given by $\sqrt{s(s-a)(s-b)(s-c)}$, where $s = (a+b+c)/2$.

10. Suppose that the incircle of $\triangle ABC$ is tangent to the sides BC, AC, and AB at the points P, Q, and R, respectively. Show that AP, BQ, and CR are concurrent.

11. Suppose the points of a geometry $\mathcal{G}$ are the points strictly inside the unit circle centred at the origin in $\mathbb{R}^2$, *i.e.* (x, y) such that $x^2 + y^2 < 1$, and the lines of the geometry are (all of) the chords of the circle. Angles and distances are measured just as in $\mathbb{R}^2$. Determine which of Euclid's five postulates hold in $\mathcal{G}$.

Part ◯.

↺. Write an original poem about geometry. *[2]*

[Total = 80]

I HOPE THAT YOU ENJOYED THIS COURSE.
ENJOY THE SUMMER!