Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2012

Assignment #α
A geometry on a cylinder
Due on Monday, 27 February, 2012.

We will define a geometry $\mathcal{G}$ of points and lines on the cylinder $x^2 + y^2 = 1$ in $\mathbb{R}^3$. The *points* of the geometry $\mathcal{G}$ are the points of the cylinder $x^2 + y^2 = 1$, *i.e.* the points (x, y, z) in three-dimensional Cartesian coordinates which satisfy the equation. The *lines* of the geometry $\mathcal{G}$ are the curves on the cylinder obtained by intersection a plane through the origin in three-dimensional space with the cylinder. (Note that every plane through the origin does indeed intersect the cylinder.) Your task will be to determine some of the basic properties of $\mathcal{G}$.

1. Does $\mathcal{G}$ satisfy Postulate I? Show it does or show that it doesn't. *[2]*

2. Give (different! :-) examples to show that it is possible for distinct lines ℓ and m of $\mathcal{G}$ to intersect in 0 or 2 points of $\mathcal{G}$. *[3]*

3. Explain why distinct lines ℓ and m of $\mathcal{G}$ cannot intersect in exactly 1 point of $\mathcal{G}$. *[2]*

4. Determine whether or not *Playfair's Axiom*,

> If P is a point not on the line ℓ, then there is a unique line m through P such that l and m do not intersect in any point.

is true in $\mathcal{G}$. *[3]*

NOTE: This extra assignment may only be used to replace your result on Assignment #1.