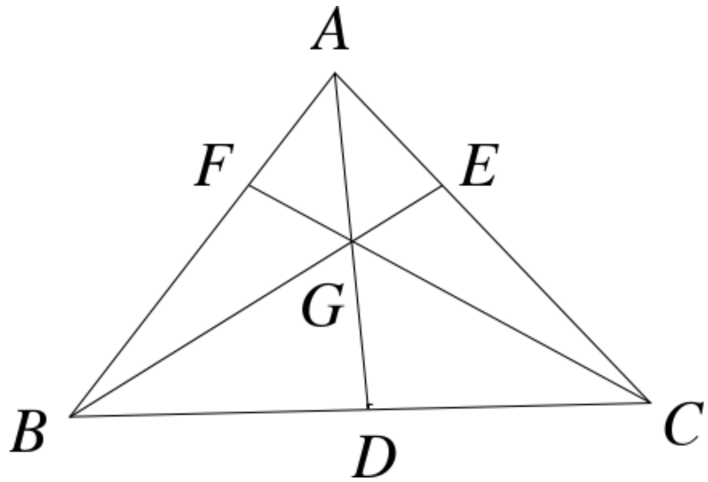
Mathematics 2260H – Geometry I: Euclidean geometry
Trent University, Winter 2012

Assignment #6
Ceva's Theorem
Due on Thursday, 1 March, 2012.

The following result appears to have been first obtained by the Arab mathematician Yusuf ibn Ahmad al-Mu'taman ibn Hud, who also served as the ruler of the Emirate of Zaragoza from 1082 to 1085. It was later rediscovered by an Italian Jesuit, Giovanni Ceva (1647-1734), who also rediscovered Menelaus' Theorem.

> Ceva's Theorem: Suppose D, E, and F are points on the sides BC, AC, and AB, respectively, of $\triangle ABC$. Then AD, BE, and CF all meet in a single point G if and only if $\dfrac{AF}{FB} \cdot \dfrac{BD}{DC} \cdot \dfrac{CE}{EA} = 1$.



1. Prove Ceva's Theorem. *[8 = 2 × 4 each for each direction]*

Hint: ($\Longrightarrow$) You may exploit the fact that the areas of two triangles with the same height are in the same proportion as their bases. Recast the product of ratios as a product of ratios of areas of subtriangles in two different ways, and from there recast is as a third product of ratios of areas of subtriangles.

($\Longleftarrow$) Let G be the intersection of AD and BE and extend CG until it intersects AB at H. Use the $\Longrightarrow$ to help show that $H = F$.

2. Use Ceva's Theorem to verify that the three *medians* of a triangle (*i.e.* the lines joining each vertex to the midpoint of the opposite side) are *concurrent* (*i.e.* meet at a single point).

Note: The point where the three medians of a triangle are concurrent is the *centroid* of the triangle. It is one of several possible "centres" of the triangle; we will encounter several others later.