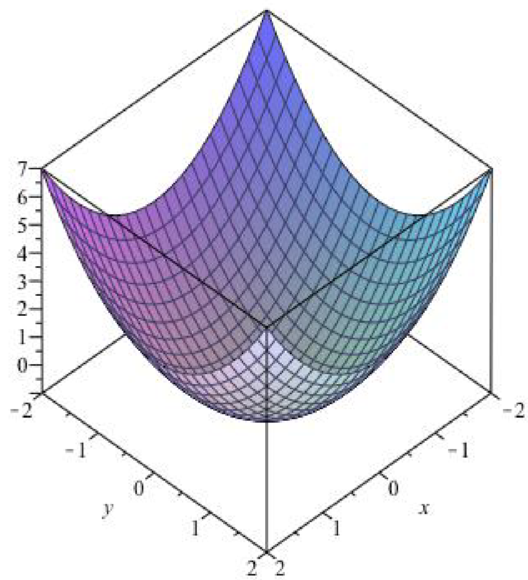
Mathematics 2260H – Geometry I: Euclidean geometry
Trent University, Winter 2012

Assignment #1
A geometry on a paraboloid
Due on Thursday, 19 January, 2012.

We will define a geometry $\mathcal{G}$ of points and lines on (the surface of) the paraboloid $z = x^2 + y^2 - 1$, part of which pictured below. (The diagram was generated using `Maple` with the command: `plot3d(x^2+y^-1,x=-2..2,y=-2..2,axes=boxed);`)



The *points* of the geometry $\mathcal{G}$ are the points of the paraboloid, *i.e.* the points (x, y, z) in three-dimensional Cartesian coordinates which satisfy the equation $z = x^2 + y^2 - 1$. The *lines* of the geometry $\mathcal{G}$ are the curves on the paraboloid obtained by intersection a plane through the origin in three-dimensional space with the paraboloid. (Note that every plane through the origin does indeed intersect the paraboloid.) Your task will be to determine some of the basic properties of $\mathcal{G}$.

1. Suppose P and Q are two distinct points of $\mathcal{G}$. Show that there is a unique line ℓ of $\mathcal{G}$ which passes through both P and Q. *[3]*

2. Give (different! :-) examples to show that it is possible for distinct lines ℓ and m of $\mathcal{G}$ to intersect in 0, 1, or 2 points of $\mathcal{G}$. *[3]*

3. Explain why distinct lines ℓ and m of $\mathcal{G}$ cannot intersect in 3 points of $\mathcal{G}$. *[1]*

4. Determine whether or not *Playfair's Axiom*,

> If P is a point not on the line ℓ, then there is a unique line m through P such that l and m do not intersect in any point.

is true in $\mathcal{G}$. *[3]*