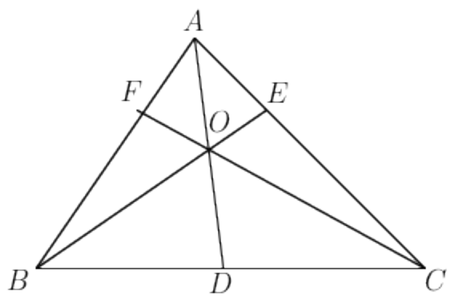
Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2011

Problem Set #9
Applications of Ceva's Theorem
Due the week of Monday, 21 March, 2011.

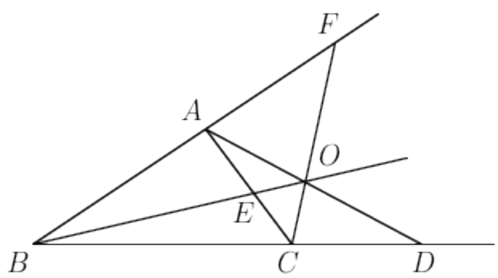
The following result appears to have been first obtained by the Arab mathematician Yusuf ibn Ahmad al-Mu'taman ibn Hud, who also served as the ruler of the Emirate of Zaragoza from 1082 to 1085. It was later rediscovered by an Italian Jesuit, Giovanni Ceva (1647-1734), who also rediscovered Menelaus' Theorem.

CEVA'S THEOREM. Suppose D, E, and F are points on the sides BC, AC, and AB, respectively, of $\triangle ABC$. Then AD, BE, and CF are concurrent (*i.e.* intersect in a common point) if and only if $\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$.

Here is a diagram with the point of concurrence, O, inside the triangle,



... and here is one with the point of concurrence outside the triangle.



1. Recall that the medians of a triangle are the line segments connecting each vertex to the midpoint of the opposite side. Use Ceva's Theorem to verify that the medians of a triangle are concurrent. *[10]*

NOTE. Recall that the point in which the medians meet is the centroid of the triangle.

2. Use Ceva's Theorem to verify that the angle bisectors of the interior angles of a triangle are concurrent. *[10]*

NOTE. Recall that the point in which the medians meet is the incentre of the triangle.