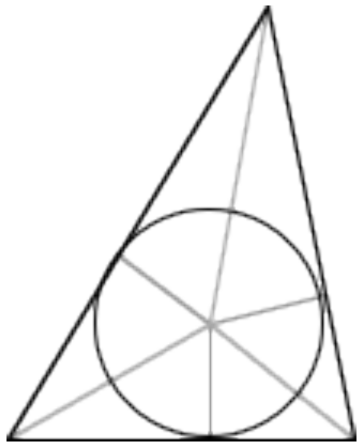
Mathematics 2260H – Geometry I: Euclidean geometry
Trent University, Winter 2011

Problem Set #8
Circles and triangles again.
Due the week of Monday, 14 March, 2011.

Definition. The *incircle* or *inscribed circle* of a triangle is the largest circle that can fit inside the triangle. The centre of the incircle is the triangle's *incentre*.



1. Give an informal argument explaining why the incircle of a triangle turns out to be the unique circle that is tangent to all three sides of the triangle. *[5]*
2. Show that the angle bisectors of the interior angles of a triangle all meet in the triangle's incentre. *[15]*

Note: The incentre is one of several "centres" of a triangle. At one time or another, we have already seen the centroid, where the medians (*i.e.* the lines joining each vertex to the midpoint of the opposite side) of the triangle all meet, and the circumcentre, where the perpendicular bisectors of the three sides all meet. Recall that the circumcentre is the center of the triangle's circumcircle, the unique circle that passes through all three of the triangles vertices. There is one other common "centre" of a triangle, the *orthocentre*, where the three altitudes (*i.e.* the perpendiculars from each vertex to the oppposite side) all meet.

It is a peculiar fact that the centroid, orthocentre, and circumcentre of a triangle which is not equilateral all lie on the same line, the *Euler line* of the triangle, as do several other points of interest for the triangle. (In an equilateral triangle, the centroid, orthocentre, and circumcentre are all the same point, so they cannot hope to determine a line.) The incentre, however, does not lie on the Euler line unless the triangle is isosceles.