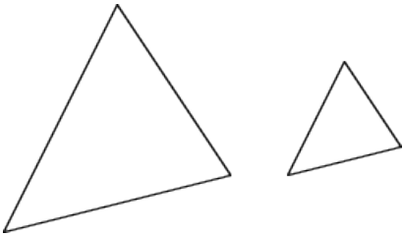
Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2011

Problem Set #6
Similarity
Due on Monday, 28 February, 2011.

Definition. Two triangles are *similar*, often written as $\triangle ABC \sim \triangle DEF$, if one is a scaled copy of the other; that is, if $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.

Note that if the triangles are congruent, then $AB = DE$, $BC = EF$, and $AC = DF$, so $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 1$. Hence congruent triangles are similar, but it should be pretty obvious that similar triangles need not be congruent:



In this assignment you will develop some of the basic properties of similar triangles. You may – and should, as necessary – assume whichever version(s) you wish of the Parallel Postulate.

1. $\triangle ABC \sim \triangle DEF$ if and only $\angle ABC = \angle DEF$, $\angle BCA = \angle EFD$, and $\angle CAB = \angle FDE$. *[8]*
2. *(Angle-Angle Similarity Criterion)* If $\angle ABC = \angle DEF$ and $\angle BCA = \angle EFD$, then $\triangle ABC \sim \triangle DEF$. *[6]*
3. *(Side-Angle-Side Similarity Criterion)* If $\angle ABC = \angle DEF$ and $\frac{AB}{DE} = \frac{BC}{EF}$, then $\triangle ABC \sim \triangle DEF$. *[6]*

NOTE: Similarity really, really, does not work the same way in non-Euclidean geometries as it does in Euclidean geometry. In particular, in both hyperbolic and spherical geometry two triangles are similar if and only if they are actually congruent.