

Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2011

Problem Set #10

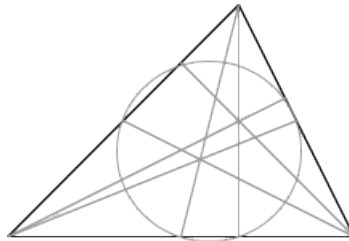
If the Nazgûl studied circles ...

Due the week of Monday, 28 March, 2011.

The various parts of the following fact were discovered by several mathematicians in the nineteenth century.

THE NINE-POINT CIRCLE. Suppose D , E , and F are the mid-points of the sides BC , AC , and AB , respectively, of $\triangle ABC$. Suppose also that AX , BY , and CZ are the three altitudes of $\triangle ABC$, which meet in the orthocentre O , and that P , Q , and R are the midpoints of AO , BO , and CO , respectively. Then the nine points D , E , F , X , Y , Z , P , Q , and R all line on a common circle, the *nine-point circle* of $\triangle ABC$.

Here is a diagram of the nine-point circle of an acute triangle, with all the points unlabelled:



The centre of the nine-point circle turns out to be on the Euler line of the triangle, halfway between the orthocentre and the circumcentre.

1. Prove the fact given above. [20]

NOTE. You may look this up, but be sure to rewrite whatever argument you find so that it can be justified on the basis of what we have learned in the course.