Mathematics 2260H – Geometry I: Euclidean geometry

TRENT UNIVERSITY, Winter 2011

Problem Set #1

Due on Monday, 17 January, 2011.

Yet another geometry!

The object of this assignment is for you to get used to doing geometrical arguments in a simplified context. Here are the axioms for an *affine plane*:

A1. Any two distinct points are both on a unique line.

A2. Given any line and any point which is not on the line, there is a unique line through the given point which does not meet the given line.

A3. There are three points which are not all on the same line.

Note that these axioms make absolutely no reference to notions like distance, angle, or even notions like betweenness. It's all points, lines, and the relation of "incidence" (*i.e.* of a point being on a line or not).

1. Verify that the Euclidean plane is an affine plane. *[4]*

2. Give an example of a finite affine plane, *i.e.* a finite configuration of "points" and "lines" that satisfies **A1**–**A3**. *[4]*

3. Using nothing but the given axioms, show that any two lines of an affine plane have the same number of points. *[5]*

4. Suppose you are given an affine plane in which every line has n points. How many points does the entire affine plane have? How many lines? Explain why! *[7]*

"Parallel lines meet at infinity!"
Euclid repeatedly, heatedly, urged.
Until he died, and so reached that vicinity:
in it he found that the damned things diverged.

A grook by *Piet Hein*.