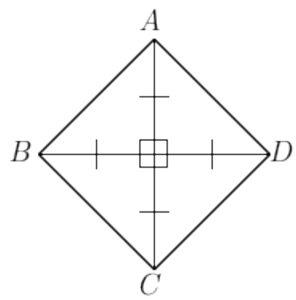
Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2011
Take-Home Final Examination
Due on Tuesday, 26 April, 2011.

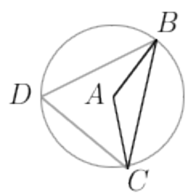
Instructions: Do both of parts **A** and **B**, and, if you wish, part $\triangle$ as well. Show all your work. You may use any sources you like, provided that you give all that contributed to your final solutions due credit, and that you adapt whatever arguments you find to work on the basis of the material covered in this course. You may also ask the instructor to clarify any of the problems, but you may not consult or work with any other person.

Part A. Do *all four* of problems **1** – **4**. *[40 = 4 × 10 each]*

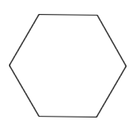
1. Suppose $ABCD$ is a quadrilateral such that the diagonals AC and BD are each other's perpendicular bisectors and that $AC = BD$. Prove that $ABCD$ is a square.



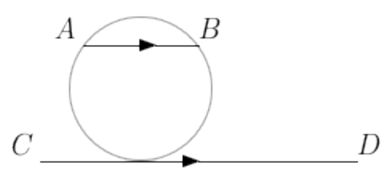
2. Suppose $\triangle ABC$ is an isosceles triangle with $AB = AC$, and D is a point on the same side of BC as A such that $\angle BAC = 2\angle BDC$. Show that A is the circumcentre of $\triangle DBC$.



3. Use Euclid's Postulates (as augmented in the textbook) to show that there is a regular hexagon, *i.e.* a polygon with six equal sides and with all six interior angles equal.



4. Suppose AB is parallel to CD. Show that there is a unique circle which passes through A and B and is tangent to the line CD.



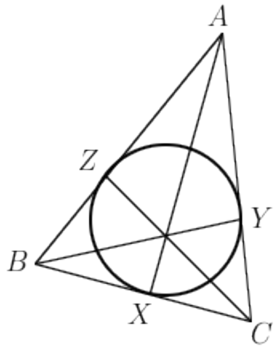
Part B. Do *any three* of problems **5** – **9**. *[30 = 3 × 10 each]*

5. Prove the angle-angle-side (AAS) congruence criterion, *i.e.* if $\angle CAB = \angle RPQ$, $\angle ABC = \angle PQR$, and $BC = QR$, then $\triangle ABC \cong \triangle PQR$.

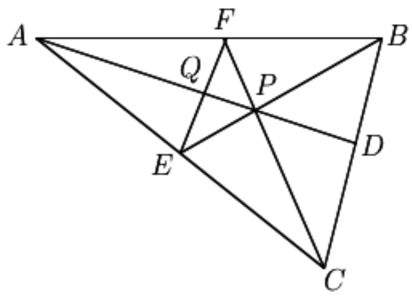
6. Prove Playfair's Postulate using Euclid's system (as augmented in the textbook), with Euclid's Postulate V replaced by the Triangle Postulate. (See Problem Set #5 for the statements of Playfair's Postulate and the Triangle Postulate.)

7. Suppose $ABCD$ is a quadrilateral. Show that there is a circle passing through all four points A, B, C, and D if and only if $\angle ABC + \angle ADC$ is a straight angle.

8. Suppose the incircle of $\triangle ABC$ is is tangent to the sides BC, AC, and AB at the points X, Y, and Z, respectively. Show that AX, BY, and CZ are concurrent.



9. Suppose D, E, and F are points on the sides BC, AC, and AB, respectively, of $\triangle ABC$ such that AD, BE, and CF are concurrent at a point P inside the triangle. Let Q be the point where AD meets EF. Show that $AQ \cdot PD = PQ \cdot AD$.



Part △.

▲. Write an original poem about geometry or mathematics in general. *[2]*

[Total = 70]

I HOPE YOU ENJOYED THE COURSE
IN SPITE OF THE VARIOUS GLITCHES.
HAVE A GOOD SUMMER!