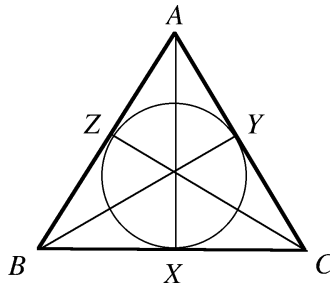


Mathematics 226H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Winter 2008

Solution to Problem Set #8

1. Suppose the Euler (*i.e.* 9-point) circle and the incircle of $\triangle ABC$ are the same circle. What kind of triangle does $\triangle ABC$ have to be? Prove it. [10]



Solution. If the 9-point circle and the incircle of $\triangle ABC$ are the same circle, then the triangle is equilateral. Consider the points at which the 9-point circle/incircle touches the sides of the triangle, say X on BC , Y on AC , and Z on AB . Then AX , BY , and CY are simultaneously altitudes, medians, and angle-bisectors of $\triangle ABC$. By Exercise 1B.2 in the text (which was also the question on Quiz #2), if the altitude from A is also the bisector of $\angle A$, then $|AB| = |AC|$. Applying 1B.2 to vertex B instead of A , we also get that $|AB| = |BC|$. Hence $\triangle ABC$ is equilateral. ■