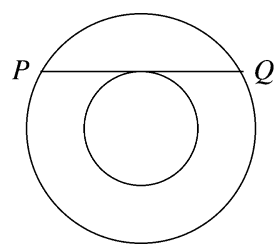
Mathematics 226H – Geometry I: Euclidean geometry
Trent University, Winter 2008

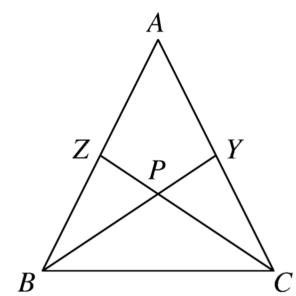
Take-Home Final Examination
Due on Friday, 25 April, 2008.

Instructions: Do all three of parts **A** – **C**, and, if you wish, part ◯ as well. Show all your work. You may use your textbooks and notes, as well as any handouts and returned work, from this and any other courses you have taken or are taking now. You may also ask the instructor to clarify the statement of any problem. However, you may not use any other sources, nor consult or work with any other person on this exam.

Part A. Do any *three* of problems **1** – **4**. *[10 each]*

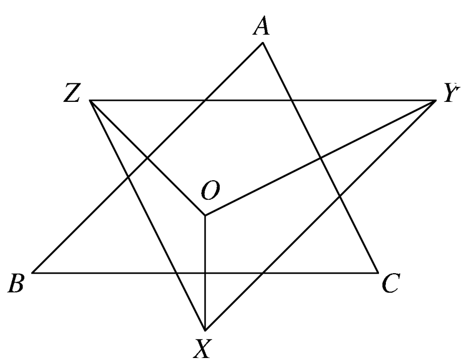
1. A chord PQ of a circle is tangent to a smaller circle with the same centre. Assuming that $|PQ| = 4cm$, find the area of the annulus between the two circles.

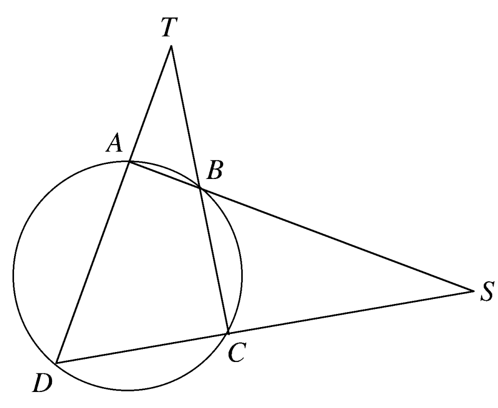




2. Let Z and Y be points on the sides AB and AC, respectively, of $\triangle ABC$, and let P be the point where BY and CZ intersect. Show that if $|BY| = |CZ|$ and $|PY| = |PZ|$, then $|AB| = |AC|$.

3. Suppose O is the circumcentre of $\triangle ABC$ and points X, Y, and Z are chosen so that BC is the perpendicular bisector of OX, AC is the perpendicular bisector of OY, and AB is the perpendicular bisector of OZ. Show that $\triangle XYZ \cong \triangle ABC$.

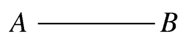


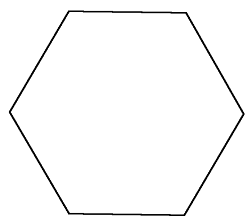


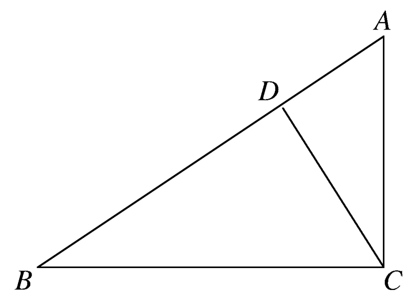
4. Suppose $ABCD$ is a cyclic quadrilateral such that the lines AB and DC intersect in the point S and the lines DA and BC intersect in the point T. Determine whether $\triangle ADS$ and $\triangle CDT$ must or must not have the same incentre.

Part B. Do any *three* of problems **5** – **9**. *[10 each]*

5. Suppose a line segment AB is given. Using Euclid's axioms, show that there is a regular hexagon with sides of length $|AB|$.

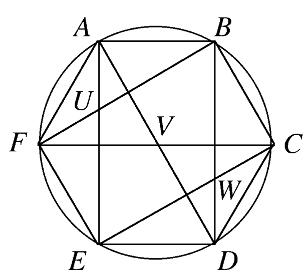


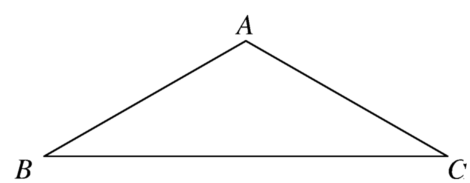




6. Suppose $\triangle ABC$ has a right angle at C and let CD be the altitude from C of this triangle. Let r, q, and p be the inradii of $\triangle ABC$, $\triangle CAD$, and $\triangle CBD$, respectively. Show that $r + q + p = |CD|$.

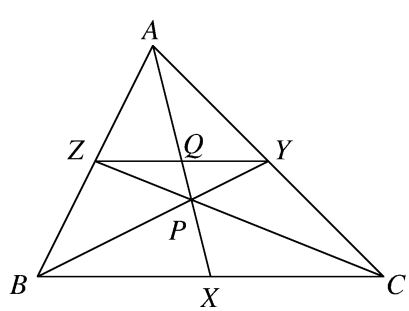
7. Suppose $ABCDEF$ is a cyclic hexagon. Let U be the intersection of AE and BF, V be the intersection of AD and CF, and W be the intersection of BD and CE. Show that U, V, and W are collinear.





8. Suppose $\triangle ABC$ has $|AB| = |AC| = 1$ and $|BC| = \sqrt{3}$, and orthocentre H. Show that $\triangle HBC$ is equilateral.

9. Suppose AX, BY, and CZ are interior Cevians of $\triangle ABC$ which are concurrent at a point P, and let Q be the point at which YZ intersects AX. Show that $|AQ| \cdot |PX| = |PQ| \cdot |AX|$.



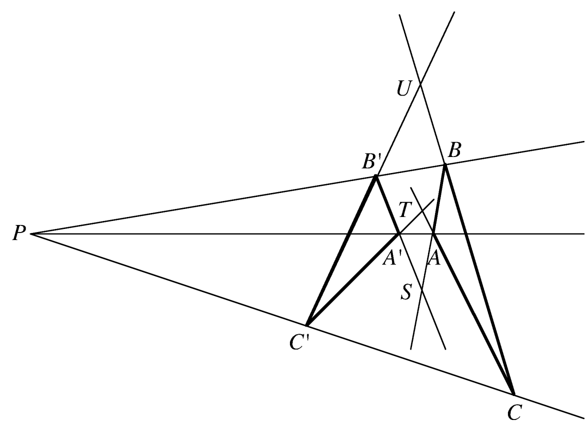
Part C. Do either *one* of problems **10 – 11**. *[10 each]*

10. Suppose $ABCD$ is a cyclic quadrilateral. Show that if we join each of its vertices to the orthocentre of the triangle formed by the other three vertices, the resulting line segments have a common midpoint M.

Draw your own picture for 10!

11. Use Pappus' Theorem to prove (the 2-D version of) Desargues' Theorem:

Suppose $\triangle ABC$ and $\triangle A'B'C'$ are positioned so that the lines AA', BB', and CC' all meet in a point P, the lines AB and $A'B'$ meet in a point S, the lines AC and $A'C'$ meet in a point T, and the lines BC and $B'C'$ meet in a point U. Then S, T, and U are collinear.



Part ◯.

⊚. Write an original poem about geometry or mathematics in general. *[2]*

[Total = 70]

I HOPE YOU ENJOYED THE COURSE!
HAVE A GREAT SUMMER!