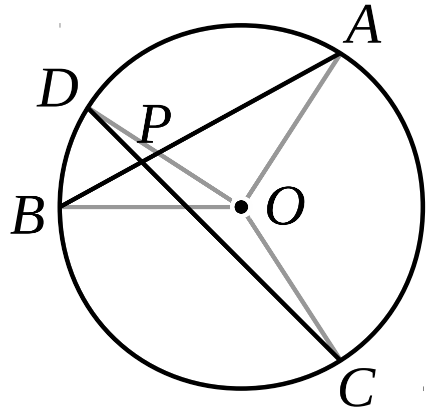
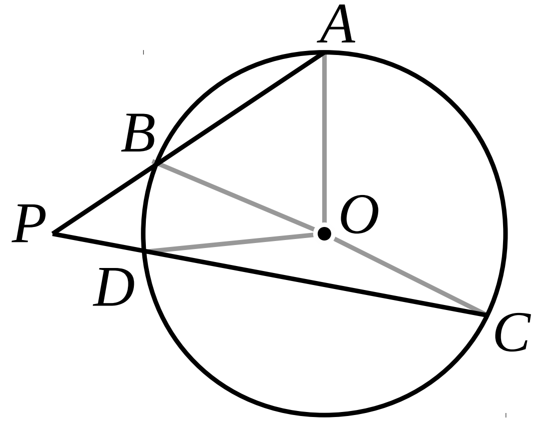
Mathematics 2260H – Geometry I: Euclidean geometry
Trent University, Fall 2018

Solutions to Assignment #8
Internal and External Angles

Suppose the chords AB and CD of a circle with centre O intersect at a point P that is not on the circle.





1. Show that $\angle APC = \frac{1}{2}(\angle AOC + \angle BOD)$ if P is inside the circle. *[5] [5]*

Solution. Suppose the chords AB and CD of a circle with centre O intersect at a point P that is inside the circle. Connect B to C and consider $\triangle BPC$. Then $\angle BPC + \angle BCP + \angle PBC = \pi\ rad$ since the sum of the interior angles of a triangle is a straight angle, so $\angle BPC = \pi - \angle PBC - \angle BCP = \pi - \angle ABC - \angle BCD = \pi - \frac{1}{2}\angle AOC - \frac{1}{2}\angle BOD$ by Proposition III-20. As $\angle APC + \angle BPC = \pi$ because A, P, and B are collinear, it now follows that

$$\angle APC = \pi - \angle BPC = \pi - \left(\pi - \frac{1}{2}\angle AOC - \frac{1}{2}\angle BOD\right) = \frac{1}{2}\angle AOC + \frac{1}{2}\angle BO\,,$$

as desired. ■

2. Show that $\angle APC = \frac{1}{2}(\angle AOC - \angle BOD)$ if P is outside the circle. *[5]*

Solution. Suppose the chords AB and CD of a circle with centre O intersect at a point P that is outside the circle. Connect B to C and consider $\triangle BPC$. Then $\angle BPC + \angle BCP = \angle ABC$ since an exterior angle of a triangle is the sum of the opposite interior angles. It follows that

$$\angle APC = \angle BPC = \angle ABC - \angle BCP = \angle ABC - \angle BCD = \frac{1}{2}\angle AOC - \frac{1}{2}\angle BOD\,,$$

as desired, with a little help from Proposition III-20 again. ■