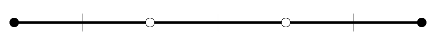
Mathematics 2260H – Geometry I: Euclidean geometry
Trent University, Fall 2018

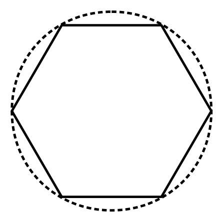
Take-Home Final Examination
Due on Friday, 14 December, 2018.

Instructions: Do both of parts − and =, and, if you wish, part ≡ as well. Show all your work. You may use your textbooks and notes, as well as any handouts and returned work, from this and any other courses you have taken or are taking now. You may also ask the instructor to clarify the statement of any problem, and use calculators or computer software to do numerical computations and to check your algebra. However, *you may not consult any other sources, nor consult or work with any other person on this exam.*

Part −. Do *all four* (4) of problems **1** – **4**. *[40 = 4 × 10 each]*

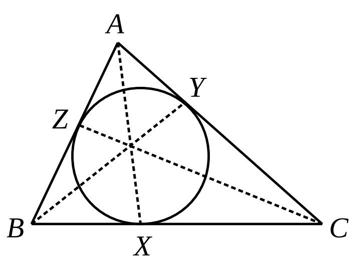
1. Use Euclid's Postulates (plus Postulates A and S) and the Propositions in Book I of the *Elements* to show that a given line segment can be divided into three equal parts.

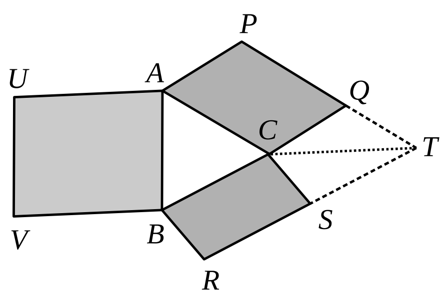




2. Suppose you are given a regular hexagon, *i.e.* a convex polygon with six equal sides and with all six interior angles equal. Show that there is a circle passing through all six vertices of the hexagon.

3. Suppose that the incircle of $\triangle ABC$ touches AB at Z, BC at X, and AC at Y. Show that AX, BY, and CZ are concurrent.





4. Suppose we are given $\triangle ABC$ and parallelograms $ACQP$ and $BCSR$. Let T be the point where PQ intersects RS. Connect C to T, and let $ABVU$ be the paralellogram such that $AU \parallel TC \parallel BV$ and $|AU| = |TC| = |BV|$. Show that the area of $ABVU$ is equal to the sum of the areas of $ACQP$ and $BCSR$. *[Another theorem of Pappus'.]*

[Parts = and ≡ are on page 2.]

Part =. Do any *four* (4) of problems **5 – 11**. *[40 = 4 × 10 each]*

Please draw the relevant diagram(s) in each problem that you choose to do!

5. Suppose that the incentre I of $\triangle ABC$ is on the triangle's Euler line. Show that the triangle is isosceles.

6. Suppose that three circles of equal radius pass through a common point P, and denote by A, B, and C the three other points where some two of these circles cross. Show that the unique circle passing through A, B, and C has the same radius as the original three circles.

7. Suppose A, B, and C are distinct points on a line ℓ, and A', B', and C' are distinct points not on ℓ such that the points $D = AB' \cap A'B$, $E = AC' \cap A'C$, and $F = BC' \cap B'C$ exist and are collinear. Show that A', B', and C' are also collinear.

8. Use Menelaus' Theorem to prove Ceva's Theorem.

9. Join each vertex of $\triangle ABC$ to the points dividing the opposite side into equal thirds and let X, Y, and Z be the points of intersection of the pairs of these lines closest to BC, AC, and AB, respectively. Show that the sides of $\triangle XYZ$ are parallel to corresponding sides of $\triangle ABC$ and that $\triangle XYZ \sim \triangle ABC$.

10. Suppose D, E, and F are points on the sides BC, AC, and AB of $\triangle ABC$, respectively, such that AD, BE, and CF all meet at a point P inside the triangle. Let Q be the point in which AD meets EF. Show that $|AQ| \cdot |PD| = |PQ| \cdot |AD|$.

11. Suppose that the convex quadrangles $ABCD$ and $A'B'C'D'$ are in perspective from the point P, with AA', BB', CC', and DD' all intersecting at P, and suppose that $S = AB \cap A'B'$, $T = BC \cap B'C'$, $U = CD \cap C'D'$, and $V = DA \cap D'A'$ all exist. Show that S, T, U, and V must all be collinear or give an example showing that this need not be the case.

[Total = 80]

Part ≡. Bonus!

∼. Write an original poem about Euclidean geometry. *[1]*

∽. Give an example of two triangles $\triangle ABC$ and $\triangle DEF$ which are *not* congruent, but which nevertheless have the same centroid G, orthocentre H, incentre I, and circumcentre O. *[1]*

Go forth and enjoy the break!