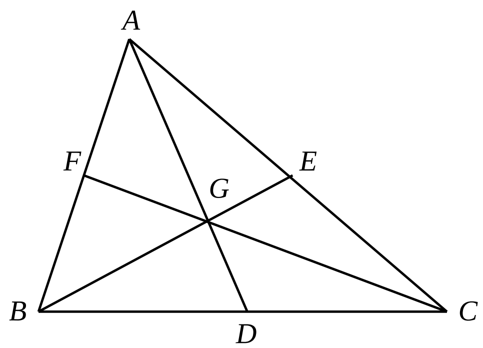
Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Fall 2018

Assignment #6
Medians

Due on Friday, 19 October.

NOTE. *You may work together on this assignment but you may not consult any sources other than the textbook and your notes.*

Suppose D, E, and F are the midpoints of sides BC, AC, and AB, respectively, of $\triangle ABC$. Then AD, BE, and CF are the *medians* of the triangle, and the point where the three medians intersect is the *centroid* of the triangle, usually denoted by G. (This is our second "centre" for a triangle after the circumcentre, which you may remember from Assignment #5.)



1. The medians divide up $\triangle ABC$ into six smaller triangles: $\triangle AGF$, $\triangle BGF$, $\triangle BGD$, $\triangle CGD$, $\triangle CGE$, and $\triangle AGE$. Show that the six smaller triangles have equal areas. *[4]*

2. Assuming that $\triangle ABC$ is cut out of a sheet of uniform thickness and density, give an informal argument explaining why the centroid G is the "balance point" of the triangle, *i.e.* the point on the triangle such that if the triangle was suspended from the point, the triangle would remain level. (Well, Archimedes, in a uniform gravity field and in the absence of other forces acting on it, and so on ... :-) *[2]*

3. Show that the three medians of a triangle are indeed *concurrent*, that is, intersect in a single point. *[4]*

Hints: There are many, sometimes very different, ways to show the medians are concurrent. Here are hints for two different methods. Feel free to find other ways ...

i. The medians of $\triangle ABC$ are also medians of its *medial triangle*, $\triangle DEF$, which is similar to $\triangle ABC$.

ii. Let G be the point where BE and CF intersect. Extend AG past G, intersecting BC at D along the way, to a point P such that $|AG| = |GP|$. Now show that $|BD| = |CD|$ by exploiting a certain parallelogram.