Assignment #10
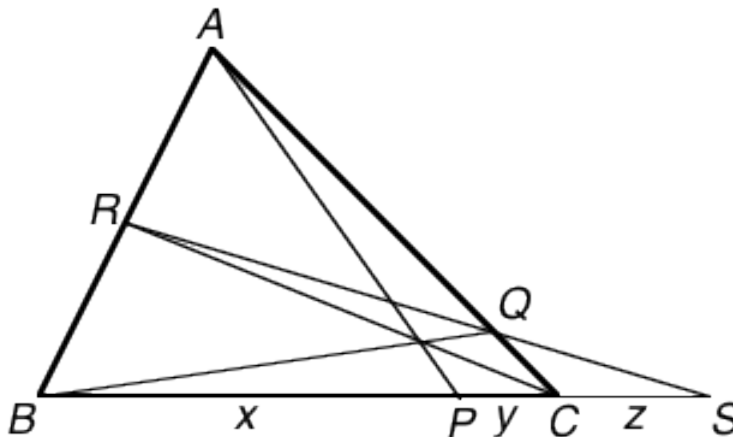**Algebra Meets Geometry**
Due on Friday, 23 November.

1. Suppose P, Q, and R are points on sides BC, AC, and AB of $\triangle ABC$ such that AP, BQ, and CR are concurrent, and suppose that RQ is extended to meet the extension of BC at S, as in the diagram below.

Show that $xz = y(x + y + z)$, where $x = |BP|$, $y = |PC|$, and $z = |CS|$. *[10]*

If the Númenoreans had been mathematicians, perhaps the rhyme of lore* Gandalf quotes to Pippin during the ride from Rohan to Gondor in the *The Lord of the Rings* would have been something like:

> *Tall ships and tall kings*
> *Three times three,*
> *What brought they from the foundered land*
> *Over the flowing sea?*
> Seven points and seven lines
> In one geometry:
> Every point met three lines,
> Every line met points three,
> Every pair of points connected,
> Every line pair intersected.
> Fano found it, Fano named it.

The poem describes the smallest finite projective plane, named after Gino Fano (1871–1952), a pioneer of finite geometry. If we were to add him to the Middle Earth "legendarium", we could make him one of the half-brothers of Fëanor (along with Fingolfin and Finarfin), the elvish craftsman who is supposed to have created the seven Palantíri or seeing stones mentioned in Tolkien's version of the rhyme of lore.

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* "Tall ships and tall kings/ Three times three,/ What brought they from the foundered land/ Over the flowing sea?/ Seven stars and seven stones/ And one white tree."