Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Fall 2018

Assignment #1
Not Plain Planes*
Due on Friday, 14 September.

Let $F = GF(3)$ be the field with three elements. That is, F is the algebraic structure whose elements are $\{0, 1, 2\}$, and which has two operations, addition and multiplication, which are done modulo 3. To be totally explicit, these operations are given by the following tables:

$$\begin{array}{cccc} + & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 2 & 0 \\ 2 & 2 & 0 & 1 \end{array} \quad \text{and} \quad \begin{array}{cccc} \cdot & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 \\ 2 & 0 & 2 & 1 \end{array}$$

It turns out that F has most of the same algebraic properties that the real numbers do, except for being seriously finite. Just as we can use the real numbers to set up coordinates (and vectors, lines, *etc.*) for familiar 2- and 3-dimensional spaces, we can use F to set up coordinates (and vectors, lines, *etc.*) for 2- and 3-dimensional finite spaces.

1. Draw a diagram of all the points and lines of F^2, the 2-dimensional Cartesian plane using F as the underlying algebraic structure. *[5]*

2. Let F^3 be the 3-dimensional space using F as the underlying structure. $\mathcal{G} = PG(2, F)$ is the geometry defined as follows:

- The *points* of $\mathcal{G}$ are the lines through the origin (*i.e.* the 1-dimensional subspaces) of F^3.
- The *lines* of $\mathcal{G}$ are the planes through the origin (*i.e.* the 2-dimensional subspaces) of F^3.
- A given point of $\mathcal{G}$ is on a given line of $\mathcal{G}$ if and only if the corresponding line of F^3 is contained in the corresponding plane of F^3.

Draw a diagram of all the points and lines of $\mathcal{G}$. *[5]*

NOTE. Neither in **1** nor in **2** should you expect all the "lines" to be straight in your diagram ...

* Plain planes fly over plane plains.