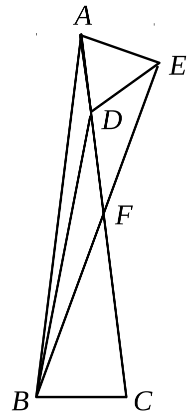
Mathematics 2260H – Geometry I: Euclidean geometry
Trent University, Fall 2016

Solutions to Assignment #3

Problems worthy of attack / Prove their worth by hitting back.*

Suppose $\triangle ABC$ has $\angle ABC = \angle ACB = 80°$ and $\angle BAC = 20°$. Let D be the point on AC between A and C such that $AD = BC$.



1. Determine $\angle ADB$ anyway you like. *[3]*

Solution. See the solution to **2** below. □

2. Determine $\angle ADB$ without using trigonometry. (You may use Postulate V or an equivalent, if you wish.) *[7]*

Solution. Construct an equilateral triangle $\triangle ADE$ such that E is on the opposite side of AC from B. Connect B to E, and let F be the point where AC intersects BE.

The sum of the interior angles of a triangle is a straight angle, *i.e.* 180°, so each of the interior angles of an equilateral triangle measures 60°. It follows that $\angle EAB = \angle EAD + \angle DAB = 60° + 20° = 80°$. Since we also have $EA = AD = BC$ and $AB = BA$, $\triangle BAE \cong \triangle ABC$ by the Side-Angle-Side congruence criterion. It follows from this congruence that $BE = AC$; since $DB = DB$, and we have $AD = ED$ by construction, it follows that $\triangle ADB \cong \triangle EDB$ by the Side-Side-Side congruence criterion.

Since $\triangle BAE \cong \triangle ABC$, $\angle ABD + \angle EBD = \angle ABE = \angle BAC = 20°$, and since $\triangle ADB \cong \triangle EDB$, $\angle ABD = \angle EBD$, so $\angle ABD = \angle EBD = 10°$. It follows that $\angle ADB = 180° - \angle BAD - \angle EBD = 180° - 20° - 10° = 150°$. ■

* A grook by Piet Hein.