Mathematics 2260H – Geometry I: Euclidean geometry

Trent University, Fall 2016

Quizzes

Quiz #1. Wednesday, 14 September. *[10 minutes]*

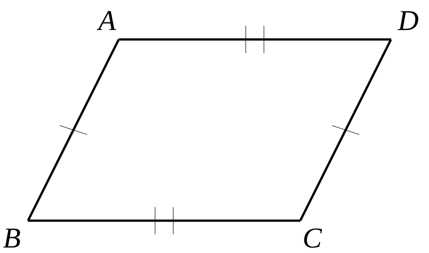
1. Suppose you are given a line (segment) AB. Using whichever you like of Postulates I–V, A, and S, as well as Propositions I-1 and I-2, show that there is a point D such that AD is three times as long as AB. *[5]*

Quiz #2. Wednesday, 21 September. *[10 minutes]*

1. Suppose that $\triangle ABC$ is equilateral and that D is the midpoint of side BC. Show that $\angle BAD = \angle CAD$. *[5]*

Quiz #3. Wednesday, 28 September. *[10 minutes]*

1. Suppose $ABCD$ is a quadrilateral such that $AB = CD$ and $BC = AD$. Show that $\angle BAD = \angle BCD$. *[5]*

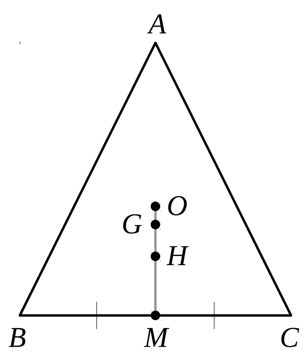


Quiz #4. Wednesday, ~~5~~ 12 October. *[10 minutes]*

1. Suppose AB is a line, P is point not on AB, and Q is a point on AB between A and B such that $\angle AQP$ is a right angle. Show that $PA > PQ$. *[5]*

Quiz #6. Wednesday, 9 November. *[10 minutes]*

1. Recall that the circumcentre O, centroid G, and orthocentre H of $\triangle ABC$ are in a straight line, which is called the Euler line of the triangle. Suppose that the midpoint M of side BC is also on this line. Show that $\triangle ABC$ must be isosceles. *[5]*



Take-home Quiz #5. Due Wednesday, 2 November. *[2 days]*
[With apologies to Prof. Tolkein ...]

If the Númenoreans had been mathematicians, perhaps the rhyme of lore* Gandalf quotes to Pippin during the ride from Rohan to Gondor in the *The Lord of the Rings* would have been something like:

Tall ships and tall kings
Three times three,
What brought they from the foundered land
Over the flowing sea?
Seven points and seven lines
In one geometry:
Every point met three lines,
Every line met points three,
Every pair of points connected,
Every line pair intersected.
Fano found it, Fano named it.

1. Draw a picture of this alternate universe Númenorean geometry, commonly known as the *Fano plane* or *Fano configuration*. *[3]*

HINT: The nicest way to draw the Fano configuration is eerily reminiscent of a symbol in the world of *Harry Potter* ...

2. Can the Fano configuration be drawn in the Euclidean plane with all of its "lines" being straight line segments of the Euclidean plane? Give an example or prove it can't be done. *[4]*

3. Is there a *quadrilateral* - four lines, no three of which pass through the same point - in the Fano plane? Give an example or show that there isn't one. *[1]*

4. Consider the following adaptation of a more famous verse† from the *Lord of the Rings*.

   One line to meet them all, at infinity to find them,
   One point to join them all and to that line to bind them,
   In planes projective where parallels die.

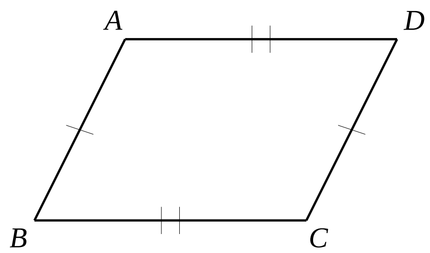
   What mathematical object(s) or operation(s) does this verse allude to? *[2]*

* "Tall ships and tall kings/ Three times three,/ What brought they from the foundered land/ Over the flowing sea?/ Seven stars and seven stones/ And one white tree."

† "One Ring to rule them all, One Ring to find them, / One Ring to bring them all and in the darkness bind them, / In the Land of Mordor where the Shadows lie."

Quiz #~~5~~ ~~6~~ 7. ~~Wednesday~~ Monday, ~~19 October~~ ~~2~~ ~~7~~ 14 November. *[10 minutes]*

1. Suppose $ABCD$ is a parallelogram, with $AB \parallel DC$ and $BC \parallel AD$ (so $AB = DC$ and $BC = AD$, too). Construct a triangle $\triangle PQR$ with area equal to $ABCD$. *[5]*

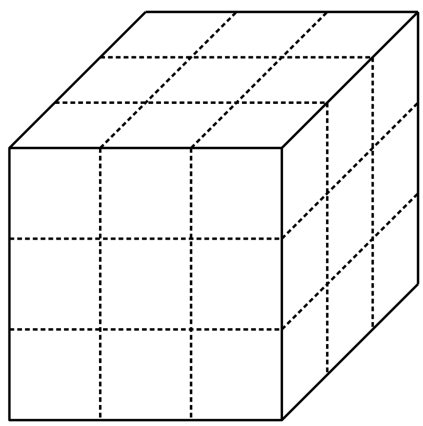


Quiz #8. Wednesday, 16 November. *[10 minutes]*

1. Suppose that the circumcentre O is on the side AC of $\triangle ABC$. Where does the orthocentre H of $\triangle ABC$ have to be? *[5]*

Take-home Quiz #9. Due Wednesday, 23 November. *[2 days]*

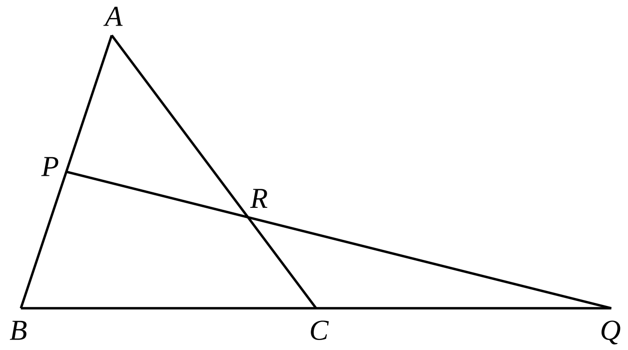
1. A carpenter, working with a circular saw, wishes to cut a wooden cube, 9 *cm* on a side, into 27 cubes, each 3 *cm* on a side. This can be done pretty easily by making six cuts through the cube, keeping the pieces together in the cube shape at each stage.



   Can the carpenter reduce the number of cuts required by rearranging the pieces after some or all of the cuts? If so, explain how to rearrange them to reduce the number of cuts; if not, explain why not. *[5]*

Quiz #10. Wednesday, 23 November. *[10 minutes]*

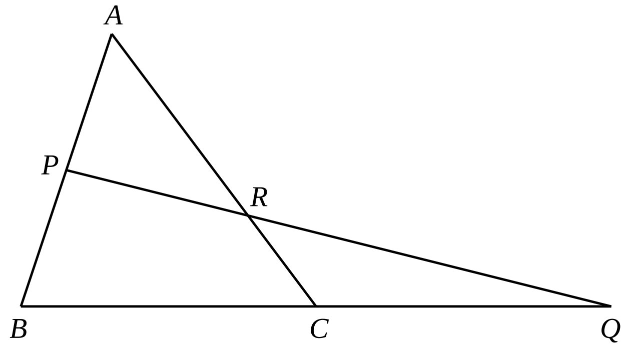
1. Suppose that we are given $\triangle ABC$, and collinear points P, Q, and R such that P is on AB between A and B, Q is on BC beyond C, and R is on AC between A and C, as in the diagram below.



Show that $\dfrac{AB}{BP} \cdot \dfrac{PQ}{QR} \cdot \dfrac{RC}{CA} = -1$. *[5]*

Quiz #10. *Alternate version. [10 minutes]*

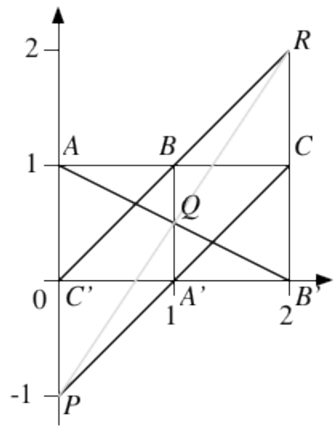
1. Suppose that we are given $\triangle ABC$, and collinear points P, Q, and R such that P is on AB between A and B, Q is on BC beyond C, and R is on AC between A and C, as in the diagram below.



Show that $\dfrac{QP}{PR} \cdot \dfrac{RA}{RC} \cdot \dfrac{CB}{BQ} = -1$. *[5]*

Quiz #11. Wednesday, 30 November. *[10 minutes]*

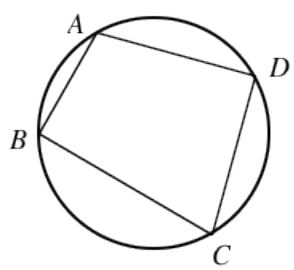
1. Suppose $A = (0, 1)$, $B = (1, 1)$, $C = (1, 2)$, $A' = (1, 0)$, $B' = (2, 0)$, and $C' = (0, 0)$ in the Cartesian plane. Let $P = AC' \cap A'C$, $Q = AB' \cap A'B$, and $R = BC' \cap B'C$.



Verify that Pappus' Theorem holds in this instance by showing directly that P, Q, and R are collinear. *[5]*

Quiz #12. Wednesday, 7 December. *[10 minutes]*

1. Suppose $ABCD$ is a convex quadrilateral inscribed in a circle.



Show that $\angle ABC + \angle CDA = 2$ right angles. *[5]*