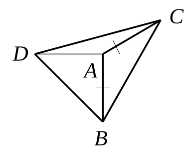
Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Fall 2016

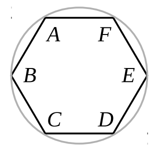
Take-Home Final Examination
Due on Friday, 16 December, 2016.

Instructions: Do both of parts $\bigcirc$ and $\triangle$, and, if you wish, part $\Box$ as well. Show all your work. You may use your textbooks and notes, as well as any handouts and returned work, from this and any other courses you have taken or are taking now. You may also ask the instructor to clarify the statement of any problem, and use calculators or computer software to do numerical computations and to check your algebra. However, *you may not consult any other sources, nor consult or work with any other person on this exam.*

Part $\bigcirc$. Do *all four* (4) of problems **1** – **4**. *[40 = 4 × 10 each]*

1. Suppose that $\triangle ABC$ is an isosceles triangle with $AB = AC$, and that D is a point on the same side of BC as A such that $\angle BAC = 2\angle BDC$. Show that $AD = AB$.





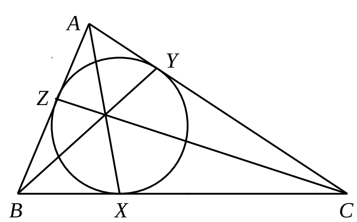
2. Suppose $ABCDEF$ is a regular hexagon, *i.e.* a convex polygon with six equal sides and with all six interior angles equal. Show that there is a circle passing through all six vertices of the hexagon.

3. An *affine plane* consists of a set of points, a set of lines (each of which is really just a set of some of the points), and the relation telling you if a points is on (*i.e.* in) a line or not, satisfying the following axioms:

I. For any two distinct points, there is exactly one line that they are both on.
II. Given a line and a point not on the line, there is a unique line including the given point that has no point in common with the given line.
III. There are three points that are not all on the same line.

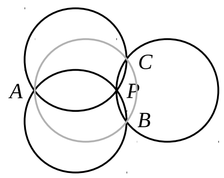
Prove that an affine plane must have at least four points and give an example of an affine plane that has exactly four points.

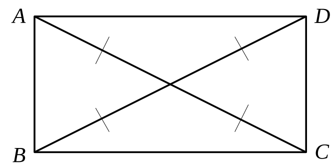
4. Suppose the incircle of $\triangle ABC$ touches AB at Z, BC at X, and AC at Y. Show that AX, BY, and CZ are concurrent.

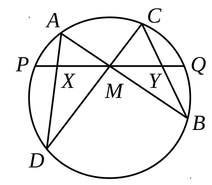


[Parts $\triangle$ and $\Box$ are on page 2.]

Part $\triangle$. Do any *four* (4) of problems **5 – 11**. *[$40 = 4 \times 10$ each]*







5. Suppose three circles of equal radius go through a common point P, and denote by A, B, and C the three other points where some two of these circles cross. Show that the unique circle through A, B, and C has the same radius as the original three circles.

6. Suppose that $ABCD$ is a convex quadrilateral such that the diagonals AC and BD have the same length and bisect each other. Show that $ABCD$ is a rectangle.

7. Suppose PQ is a chord of a circle, M is the midpoint of PQ, AB and CD are two other chords of the circle that pass through M, and suppose AD and BC meet PQ at X and Y respectively. Show that M is the midpoint of XY.

8. Assuming Postulates I through IV, A, and S, and that the sum of the interior angles of any triangle is two right angles, show that whenever P is a point not on a line ℓ, there is one and only one line through P that is parallel to ℓ.

9. Suppose that the incentre, I, of $\triangle ABC$ is on the triangle's Euler line. Show that the triangle is isosceles.

10. Suppose A, B, and C are distinct points on a line ℓ, and A', B', and C' are distinct points not on ℓ such that the points $D = AB' \cap A'B$, $E = AC' \cap A'C$, and $F = BC' \cap B'C$ exist and are collinear. Show that A', B', and C' are also collinear.

11. Suppose that the convex quadrangles $ABCD$ and $A'B'C'D'$ are in perspective from the point P, with AA', BB', CC', and DD' all intersecting at P, and suppose that $S = AB \cap A'B'$, $T = BC \cap B'C'$, $U = CD \cap C'D'$, and $V = DA \cap D'A'$ all exist. Show that S, T, U, and V must all be collinear or give an example showing that they need not be collinear.

[Total = 80]

Part $\square$. Bonus!

$\odot$. Write an original poem about Euclidean geometry. *[1]*

$\ominus$. A curve consists of all of the points (x, y) in the Cartesian plane such that the sum of the distances from (x, y) to $(1, 0)$ and to $(-1, 0)$ is 4. Find an equation for the curve that does not employ square roots. *[1]*

I HOPE THAT YOU ENJOYED THIS COURSE. NOW ENJOY THE BREAK!