Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Fall 2016

Assignment #2
Congruence, etc.
Due on Wednesday, 21 September.

Recall that triangles $\triangle ABC$ and $\triangle DEF$ are *congruent*, written as $\triangle ABC \cong \triangle DEF$, if all the corresponding sides and angles are equal. That is, $\triangle ABC \cong \triangle DEF$ exactly when $|AB| = |DE|$, $|AC| = |DF|$, $|BC| = |EF|$, $\angle ABC = \angle DEF$, $\angle BCA = \angle EFD$, and $\angle CAB = \angle FDE$. Informally, this means that you can place $\triangle ABC$ over $\triangle DEF$ (possibly needing to flip it over) so that A is exactly on D, B is exactly on E, and C is exactly on F; *i.e.* so that $\triangle ABC$ exactly covers $\triangle DEF$.

Similarly – *cough, cough* – triangles $\triangle ABC$ and $\triangle DEF$ are *similar*, written as $\triangle ABC \sim \triangle DEF$, if all the corresponding angles are the same. That is, $\triangle ABC \sim \triangle DEF$ exactly when $\angle ABC = \angle DEF$, $\angle BCA = \angle EFD$, and $\angle CAB = \angle FDE$. That is, similar triangles have the same shape, but not necessarily the same size.

We will mainly be concerned with the congruence and similarity of triangles, but the definitions can obviously be extended to polygons with more sides, and to two-dimensional shapes in general.

1. Show that congruence implies similarity for triangles in the Euclidean plane, *i.e.*
$$\triangle ABC \cong \triangle DEF \Longrightarrow \triangle ABC \sim \triangle DEF,$$
but not the other way around. *[1]*

Since we haven't yet developed all the Euclidean tools needed, you may, if you wish, use trigonometry and the fact that the interior angles of a triangle sum to two right angles (or one straight angle, or π *rad*, or $180°$, or ...) to help do the following problems.

2. Prove the Side-Angle-Side (SAS) similarity criterion for triangles, *i.e.* if $\angle ABC = \angle DEF$ and $\dfrac{|AB|}{|DE|} = \dfrac{|BC|}{|EF|}$, then $\triangle ABC \sim \triangle DEF$. *[3]*

3. Suppose P and Q are the midpoints of sides AB and AC in $\triangle ABC$. Show that $\triangle ABC \sim \triangle APQ$ and $|BC| = 2|PQ|$. *[3]*

Congruence also has a connection with linear algebra via *rigid motions* in the plane: *translations*, which slide points a fixed distance in the same direction, *rotations*, which revolve points by some fixed angle around a fixed point, and *reflections*, which swap points with their mirror images across some fixed line.

4. Suppose $\triangle ABC \cong \triangle DEF$. Explain, informally, but as completely as you can, why one can move $\triangle ABC$ so that it exactly covers $\triangle DEF$ by no more than one translation, followed by no more than one rotation, followed by no more than one reflection. *[3]*