

Mathematics 2260H – Geometry I: Euclidean geometry

TRENT UNIVERSITY, Fall 2016

Assignment #1

The Moulton plane

Due on Wednesday, 14 September.

The *affine plane of order 4* is an example of a plane geometry that is not the familiar Euclidean plane. One way to define it is to start with the field of four elements, where the operations of addition and multiplication work as given by the tables

$$\begin{array}{cccc} + & 0 & 1 & a & b \\ 0 & 0 & 1 & a & b \\ 1 & 1 & 0 & b & a \\ a & a & b & 0 & 1 \\ b & b & a & 1 & 0 \end{array} \quad \text{and} \quad \begin{array}{cccc} \cdot & 0 & 1 & a & b \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & a & b \\ a & 0 & a & b & 1 \\ b & 0 & b & 1 & a \end{array}$$

(note that $a^2 = b = a + 1$ and $b^2 = a = b + 1$), and then define points and lines using Cartesian-style coordinates, just like we use the real numbers and the usual operations on them to build the Cartesian plane.

1. Draw a picture of the affine plane of order 4. [3]
2. Determine as fully you can which of Euclid's five Postulates, plus Postulates S and A from the textbook (pp. 39-41), make sense and are satisfied in the affine plane of order 4. [7]

Little Jack Horner
Sat in the corner
Trying to work out π .
He said 'It's minus the logarithm
Of minus one to the i .'

Cribbed from *Seven Years of Manifold 1968-1980*.