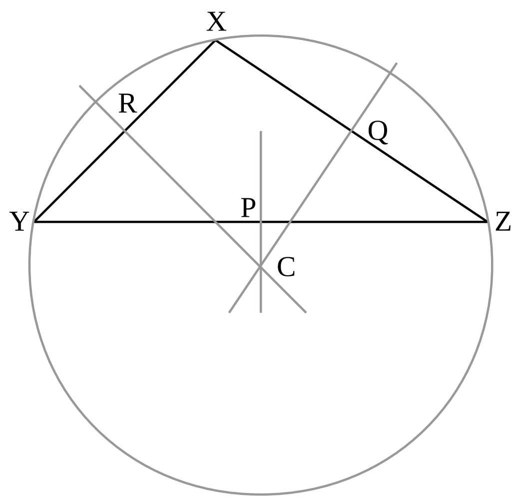
Mathematics 2260H – Geometry I: Euclidean geometry
Trent University, Fall 2015

Assignment #4
Centres and circles for triangles
Due on Monday, 9 November, 2015.

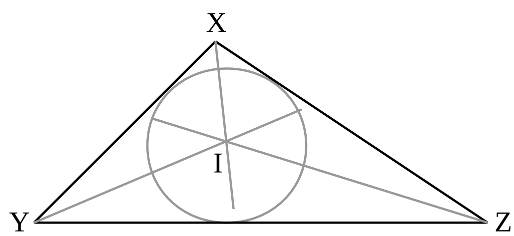
Before tackling these problems, it might be a good idea to review the results in the textbook about when a point is equidistant from two given points or two given lines. These results are implicit in some of Euclid's Propositions from Book I, especially I-10 through I-12, but are explicitly stated in the textbook.

1. Show that the three perpendicular bisectors of the sides of a triangle are concurrent [*i.e.* meet at a single point], and that this point is the centre of a circle that passes through all three vertices of the triangle. *[5]*



Note: The point in which the perpendicular bisectors meet is called the *circumcentre* of the triangle and the circle is called the *circumcircle* of the triangle.

2. Show that three internal angle bisectors of a triangle are concurrent and that this point is the centre of a circle that touches each of the sides of the triangle at a single point. *[5]*



Note: The point where the three internal angle bisectors of a triangle are concurrent is called the *incentre* of the triangle and the circle is called the *incircle* of the triangle.

Note: This gives us three centres for a triangle so far, starting with the centroid from Assignment #3. It is traditional to denote the centroid by G, the circumcentre by C, and the incentre by H. Still to come are the *orthocentre*, where the three altitudes meet, usually denoted by H, and the centre of the nine-point circle of the triangle, usually denoted by N. There are *thousands* of other definitions of centres of a triangle ...