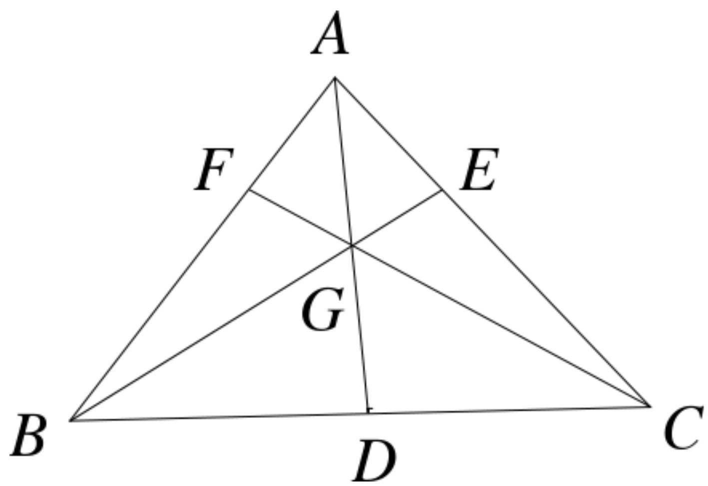
Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Fall 2015
Assignment #3
Areas $\Longrightarrow$ Intersections
Due on Monday, 19 October, 2015.

1. Show that a triangle with a base of length b and height h has area $\frac{1}{2}bh$. *[1]*

NOTE: You may use any correct method to prove **1**.

The following result appears to have been first obtained by the Arab mathematician Yusuf ibn Ahmad al-Mu'taman ibn Hud, who also served as the ruler of the Emirate of Zaragoza from 1082 to 1085. It was later rediscovered by an Italian Jesuit, Giovanni Ceva (1647-1734). The fact that someone else got there first doesn't seem to have been known until most of Yusuf ibn Hud's major work, the *Kitab al-Istikmal*, was reassembled from fragments of surviving manuscripts in the 1980s.

CEVA'S THEOREM: Suppose D, E, and F are points on the sides BC, AC, and AB, respectively, of $\triangle ABC$. Then AD, BE, and CF all meet in a single point G if and only if $\dfrac{|AF|}{|FB|} \cdot \dfrac{|BD|}{|DC|} \cdot \dfrac{|CE|}{|EA|} = 1$.



2. Prove Ceva's Theorem. *[8 = 2 × 4 each for each direction]*

HINT: ($\Longrightarrow$) Exploit the fact that the areas of two triangles with the same height are in the same proportion as their bases. Recast the product of ratios as a product of ratios of areas of subtriangles in two different ways, and from there recast it as a third product of ratios of areas of subtriangles.

($\Longleftarrow$) Let G be the intersection of AD and BE and extend CG until it intersects AB at H. Use the $\Longrightarrow$ to help show that $H = F$.

3. Use Ceva's Theorem to verify that the three *medians* of a triangle [the lines joining each vertex to the midpoint of the opposite side] are *concurrent* [meet at a single point]. *[1]*

NOTE: The point where the three medians of a triangle are concurrent is the *centroid* of the triangle. It is one of several possible "centres" of the triangle; we will encounter several others later.