Mathematics 2260H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Fall 2015

Assignment #2
Triangles and Similarity
Due on Monday, 5 October, 2015.

Recall that triangles $\triangle ABC$ and $\triangle DEF$ are *congruent*, written as $\triangle ABC \cong \triangle DEF$, if all the corresponding sides and angles are equal. That is, $\triangle ABC \cong \triangle DEF$ exactly when $|AB| = |DE|$, $|AC| = |DF|$, $|BC| = |EF|$, $\angle ABC = \angle DEF$, $\angle BCA = \angle EFD$, and $\angle CAB = \angle FDE$.

Similarly – cough – triangles $\triangle ABC$ and $\triangle DEF$ are *similar*, written as $\triangle ABC \sim \triangle DEF$, if all the corresponding angles are the same. That is, $\triangle ABC \sim \triangle DEF$ exactly when $\angle ABC = \angle DEF$, $\angle BCA = \angle EFD$, and $\angle CAB = \angle FDE$. As we shall see, similar triangles are exactly the same shape, but not necessarily the same size.

We will mainly be concerned with triangles when dealing with congruence and similarity, but the definitions can be extended in obvious ways to polygons with more sides, and to two-dimensional shapes in general.

1. Show that congruence implies similarity for triangles in the Euclidean plane, *i.e.*
$$\triangle ABC \cong \triangle DEF \Longrightarrow \triangle ABC \sim \triangle DEF,$$
but not the other way around. *[2]*

Since we haven't yet developed all the Euclidean tools needed, you may, if you wish, use trigonometry and the fact that the interior angles of a triangle sum to two right angles (or one straight angle, or π *rad*, or $180°$, or ...) to help do the following problems.

2. Prove the Angle-Side-Angle (ASA) congruence criterion for triangles, *i.e.* if $\angle ABC = \angle DEF$, $|BC| = |EF|$, and $\angle BCA = \angle EFD$, then $\triangle ABC \cong \triangle DEF$. *[2]*

3. Prove the Angle-Angle (AA) similarity criterion for triangles, *i.e.* if $\angle ABC = \angle DEF$ and $\angle BCA = \angle EFD$, then $\triangle ABC \sim \triangle DEF$. *[2]*

4. Prove the Side-Angle-Side (SAS) similarity criterion for triangles, *i.e.* if $\angle ABC = \angle DEF$ and $\dfrac{|AB|}{|DE|} = \dfrac{|BC|}{|EF|}$, then $\triangle ABC \sim \triangle DEF$. *[2]*

5. Suppose P and Q are the midpoints of sides AB and AC in $\triangle ABC$. Show that $\triangle ABC \sim \triangle APQ$ and $|BC| = 2|PQ|$. *[2]*