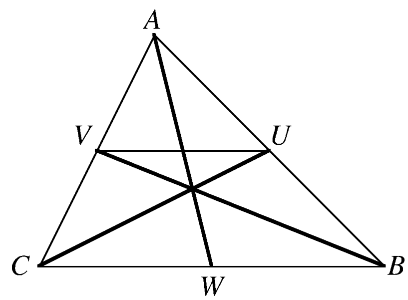
Mathematics 226H – Geometry I: Euclidean geometry
TRENT UNIVERSITY, Fall 2006

Solutions to Problem Set #3

1. (Exercise 1H.2) Show that the three medians of a triangle go through a common point [the *centroid* of the triangle]. *[5]*

Hint: Use Problem 1.30.

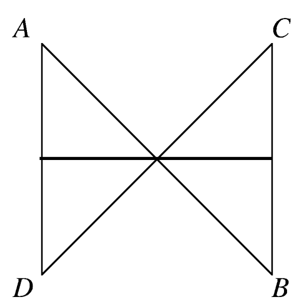
Solution. We will use both Problem 1.30 and Corollary 1.31. Given $\triangle ABC$, let U, V, and W be the midpoints of AB, AC, and BC, respectively. (Hence AW, BV, and CU are the three medians of the triangle.)



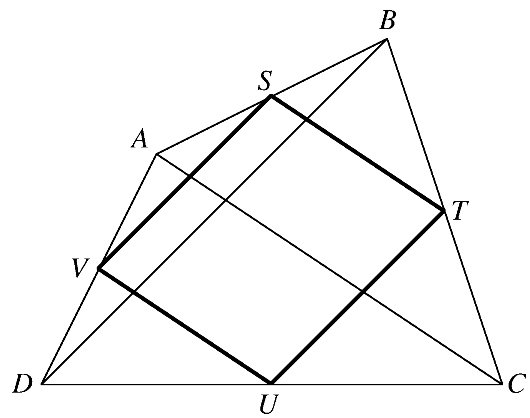
Since U and V are the midpoints of AB and AC, respectively, $UV \parallel BC$ by Corollary 1.31. By Problem 1.30, it follows that the intersection of BV and CU lies on the median of $\triangle ABC$ from point A, *i.e.* on AW. Hence the three medians, BV, CU, and AW, meet at a common point, as required. ■

2. (Exercise 1H.3) Let $ABCD$ be an arbitrary parallelogram. Show that the midpoints of the four sides are the vertices of a parallelogram. *[5]*

Note: You were allowed to assume that the quadrilateral $ABCD$ is convex. The result is true for non-convex quadrilaterals whose sides don't cross each other, but it can fail for "degenerate" quadrilaterals such as the one below:



Solution. Let S, T, U, and V be the midpoints of AB, BC, CD, and DA, respectively, as in the diagram below. We need to show that $STUV$ is a parallelogram.



Since S and T are the midpoints of sides AB and BC, respectively, of $\triangle ABC$, it follows from Corollary 1.31 that $ST \parallel AC$ and $|ST| = \frac{1}{2}|AC|$. Similarly, since U and V are the midpoints of sides CD and DA, respectively, of $\triangle CDA$, it follows from Corollary 1.31 that $UV \parallel AC$ and $|UV| = \frac{1}{2}|AC|$. Hence $ST \parallel UV$ and $|ST| = |UV|$, so quadrilateral $STUV$ is a parallelogram by Theorem 1.8. ■