Mathematics 226H – Geometry I: Euclidean geometry
Trent University, Fall 2006

Solutions to Problem Set #1

Please note that these solutions do not include any diagrams. To make sure you understand how they work, draw the diagrams for yourself, especially for problem **2**.

1. (Exercise 1B.3) If the altitude from vertex A in $\triangle ABC$ is also a median, show that $|AB| = |AC|$. *[5]*

Solution. Let P be the point at which the altitude from A intersects BC. Since AP is an altitude, $\angle APB = \angle APC = 90°$, and because AP is also a median, $|BP| = |CP|$. As we also have $|AP| = |AP|$ (for free!), $\triangle APB \cong \triangle APC$ by the side-angle-side criterion. It follows that the corresponding sides remaining are of the same length, *i.e.* $|AB| = |AC|$. ∎

2. (Exercise 1B.9) Suppose Y and Z are points on the sides AC and AB of $\triangle ABC$ respectively, and P is the intersection point of BY amd CZ. Assume that $|AB| = |AC|$ and $|BY| = |CZ|$. Is it necessarily true that $|PY| = |PZ|$? *[5]*

Solution. It is *not* necessarily true that $|PY| = |PZ|$. A counterexample can be constructed as follows:

Suppose $\triangle ABC$ is isosceles with $|AB| = |AC|$. Let Q and R be the points at which the altitudes from C and B intersect AB and AC respectively. Note that $|CQ| = |BR|$. (Why?) Choose Y to be any point on RC (different from both R and C) such that $|RY| < |QA|$, and let Z be the point on AQ for which $|QZ| = |RY|$. Observe that it follows that $\triangle YBR \cong \triangle ZCQ$ by the side-angle-side criterion, and hence $|CZ| = |BY|$. Let P be the point at which CZ and BY intersect. It should be pretty obvious from the diagram (which you should have been drawing as you were reading this) that $|PZ| > |PY|$. (You get to prove that last, if you feel up to it!) ∎

3. (Exercise 1D.3) Prove that the diagonals of a rectangle are equal. *[5]*

Solution. Suppose $ABCD$ is a rectangle. Then, by definition, $|AB| = |CD|$, $|AD| = |BC|$, and $\angle ABC = \angle DCB = 90°$ (among other facts). It follows that $\triangle ABC \cong \triangle DCB$ by the side-angle-side criterion. It follows that the remaining corresponding sides are of the same length, *i.e.* $|AC| = |BD|$. ∎