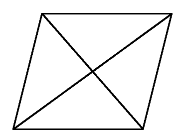
**Mathematics 226H – Geometry I: Euclidean geometry**
TRENT UNIVERSITY, Fall 2006

**Take-Home Final Examination**
*Due on Friday, 22 December, 2006.*

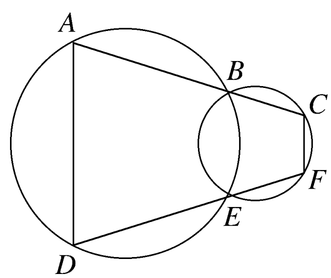
**Instructions:** Do all three of parts **A** – **C**, and, if you wish, part □ as well. Show all your work. You may use your textbooks and notes, as well as any handouts and returned work, from this and any other courses you have taken or are taking now. You may also ask the instructor to clarify the statement of any problem. However, you may not use any other sources, nor consult or work with any other person on this exam.

**Part A.** Do any *three* of problems **1** – **4**. *[10 each]*

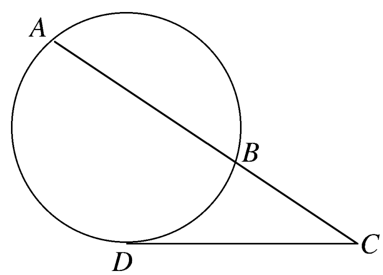
**1.** Prove that a parallelogram having perpendicular diagonals is a rhombus.



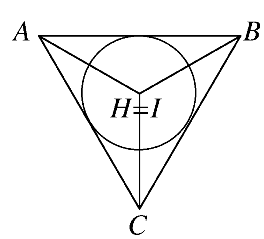
**2.** Two circles intersect at points $B$ and $E$. $A$ and $D$ on the first circle, and $C$ and $F$ on the second circle, are points such that $A$, $B$, $C$ are collinear and $D$, $E$, $F$ are collinear. Show that $AD \parallel CF$.




**3.** Suppose $A$, $B$, and $D$ are points on a circle and $C$ is a point on the line $AB$ outside the circle so that $CD$ is a tangent to the circle. Verify that $|AC| \cdot |BC| = |CD|^2$.

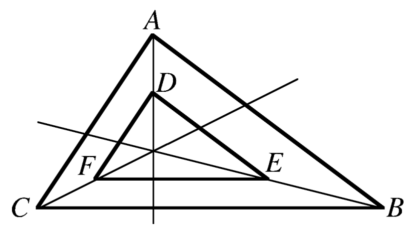



**4.** Show that if the orthocentre and incentre of $\triangle ABC$ are the same point, then the triangle is equilateral.

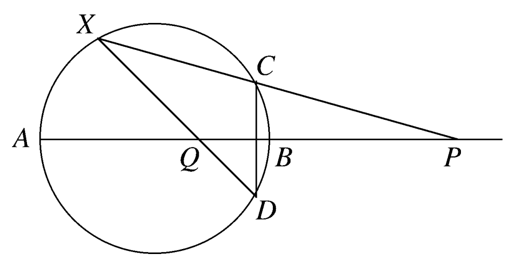


**Part B.** Do any *two* of problems **5 – 8**. *[10 each]*

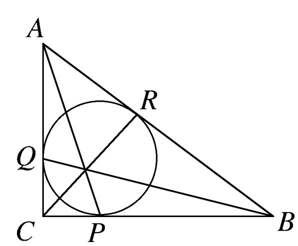
**5.** Suppose $\triangle DEF$ lies inside $\triangle ABC$, with $AB \parallel DE$, $AC \parallel DF$, and $BC \parallel EF$. Prove that the lines $AD$, $BE$, and $CF$ are concurrent.



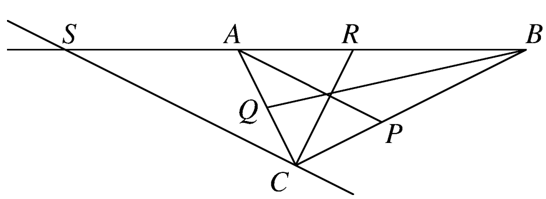

**6.** Suppose $AB$ is a diameter of a circle and $P$ a point on the line $AB$ outside the circle. Given a chord $CD$ of the circle which is perpendicular to $AB$, let $X$ be the intersection (other than $C$) of the line $PC$ with the circle, and let $Q$ be the intersection of $AB$ with $XD$. Show that $Q$ does not depend on the particular choice of the chord $CD$.




**7.** A circle inscribed in $\triangle ABC$ is tangent to $AB$ at $R$, to $AC$ at $Q$, and to $BC$ at $P$, respectively. Prove that $AP$, $BQ$, and $CR$ are concurrent.

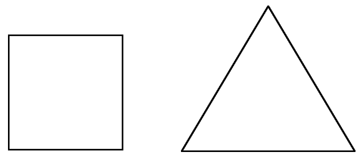



**8.** Suppose $AP$, $BQ$, and $CR$ are the angle bisectors of $\triangle ABC$, and suppose that $S$ is a point on the line $AB$ such that $CS$ is perpendicular to $CR$. Show that $P$, $Q$, and $S$ are collinear.

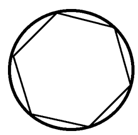


**Part C.** Do any *two* of problems **9 – 11**. *[10 each]*

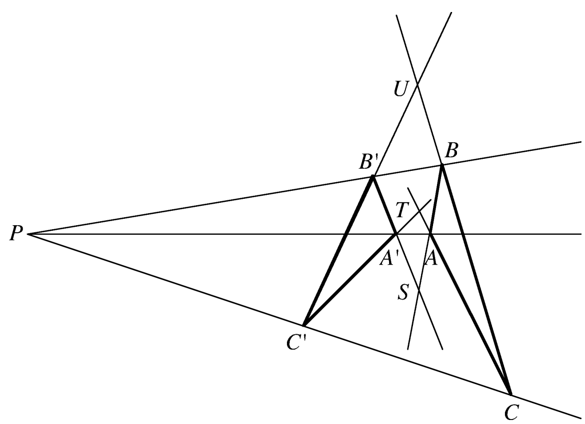
**9.** Show that one can use a ruler and compass to construct an equilateral triangle equal in area to a given square.



**10.** Show that one can use a ruler and compass to inscribe a regular hexagon in a given circle.



**11.** Suppose $\triangle ABC$ and $\triangle A'B'C'$ are positioned in such a way that the lines $AA'$, $BB'$, and $CC'$ all meet in a point $P$, the lines $AB$ and $A'B'$ meet in a point $S$, the lines $AC$ and $A'C'$ meet in a point $T$, and the lines $BC$ and $B'C'$ meet in a point $U$. Use Pappus' Theorem to help show that $S$, $T$, and $U$ are collinear.




**Part □.**

△. Write an original poem about geometry or mathematics in general. *[2]*

*[Total = 70]*

I HOPE YOU ENJOYED THE COURSE!
HAVE A NICE BREAK!