Mathematics 2200H – Mathematical Reasoning
TRENT UNIVERSITY, Fall 2025

Assignment #7
The Linear Order on $\mathbb{Z}$
Due on Friday, 31 October.[*]

Recall that a (strict) linear order on a set A, let's denote it by $\lhd$, is a binary relation satisfying the following conditions:

1. *Irreflexivity:* For all $a \in A$, it is not the case that $a \lhd a$.
2. *Transitivity:* For all $a, b, c \in A$, if $a \lhd b$ and $b \lhd c$, then $a \lhd c$.
3. *Trichotomy:* For all $a, b \in A$, exactly one of $a \lhd b$, $a = b$, or $b \lhd a$, is true.

Recall also that we defined the integers to be the set of equivalence classes of the equivalence relation $\sim$ on $\mathbb{N} \times \mathbb{N} = \{ (a,b) \mid a, b \in \mathbb{N} \}$ given by $(a,b) \sim (c,d) \iff a + d = b + c$. The equivalence class of (a,b) is then $[(a,b)]_\sim = \{ (c,d) \in \mathbb{N} \times \mathbb{N} \mid (a,b) \sim (c,d) \}$ and the set of integers is $\mathbb{Z} = \left\{ [(a,b)]_\sim \mid a, b \in \mathbb{N} \right\}$.

We can define the usual linear order on the integers in several ways. Your task, should you choose to accept it, is to ...

1. Give a formal definition of the linear order, let's call it $<_\mathbb{Z}$, on the integers. *[5]*

2. Show that $<_\mathbb{Z}$ is indeed a linear order on $\mathbb{Z}$. You may assume that we know everything you might need to know about the natural numbers to execute your proof. *[5]*

[*] Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to `sbilaniuk@trentu.ca` as soon as you can,